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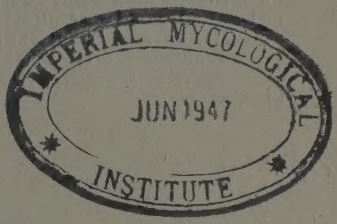

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MATHEMATICS

ON HERMITIAN OPERATORS WITH DEFICIENCY INDICES EQUAL TO ONE. II

By M. KREIN

(Communicated by S. N. Bernstein, Member of the Academy, 20. XI. 1943)

1. Retaining the same denotations and definitions as in the foregoing note (1), we formulate in the first line the following

Theorem 1. *If the u-function $f_u(z)$ corresponding to an element $f \in \mathfrak{H}$ is holomorphic on the real interval $\Delta=(a, b)$, then for any spectral function E_t of the operator A*

$$E_\Delta f = \int_a^b f_u(t)\, dE_t u \quad (E_\Delta = E_b - E_a)$$

From Theorem 1 immediately follows

Theorem 2. *If the u-functions $g_u(z)$ and $f_u(z)$ corresponding to elements $g, f \in \mathfrak{H}$ are holomorphic on the whole real axis, then for any function $\sigma \in V_u$*

$$(g, f) = \int_{-\infty}^{\infty} g_u(t)\, \overline{f_u(t)}\, d\sigma(t)$$

2. We shall norm every bounded non-decreasing function $\sigma(t)$ $(-\infty < t < \infty)$ in such a way that

$$\sigma(t) = \sigma(t-0) \quad (-\infty < t < \infty), \quad \sigma(-\infty) = 0$$

With every such function $\sigma(t)$ we shall associate the complex Hilbert space $L_\sigma^{(2)}$ consisting of all σ-measurable and σ-integrable with their squares functions $f(t)$ of the real argument t $(-\infty < t < \infty)$.

Theorem 2 shows that the set $\mathfrak{R}_u$ of all holomorphic on the real axis functions $f(z) \in \mathfrak{H}_u$ is a certain subspace of $L_\sigma^{(2)}$ $(\sigma \in V_u)$ and that the metrics established in $\mathfrak{R}_u$ as a part of $L_\sigma^{(2)}$ does not depend on the choice of $\sigma \in V_u$. This metrics coincides with that which is induced in $\mathfrak{R}_u$ as a part of $\mathfrak{H}_u$ by the algebraic isomorphism $f \longleftrightarrow f_u(z)$ between $\mathfrak{H}_u$ and the metric space $\mathfrak{H}$.

Assuming in $\mathfrak{H}_u$ the metrics thus induced, consider the case when $\mathfrak{R}_u$ is dense in $\mathfrak{H}_u$.

Theorem 3. *If the set $\mathfrak{R}_u$ of all holomorphic on the real axis functions $f(z) \in \mathfrak{H}_u$ is dense in $\mathfrak{H}_u$, then the set V_u consists of those and only those bounded non-decreasing functions $\sigma(t)$ $(-\infty < t < \infty)$ for which $(g, f) = \int_{-\infty}^{\infty} g_u(t)\, \overline{f_u(t)}\, d\sigma(t)$ for any $g_u(z)$, $f_u(z) \in \mathfrak{R}_u$. Hereby $\sigma \in V_u$ will be canonic if and only if $\mathfrak{R}_u$ is*

dense in $L_\sigma^{(2)}$. *This condition being satisfied, an isometric correspondence may be established between* $L_\sigma^{(2)}$ *and* $\mathfrak{H}_u$ (*and consequently between* $L_\sigma^{(2)}$ *and* $\mathfrak{H}$) *as complete closures of* $\mathfrak{R}_u$. *In virtue of this correspondence the operator of multiplying by the argument in* $L_\sigma^{(2)}$ *induces in* $\mathfrak{H}$ *an operator which is a self-adjoint extension of the operator* A.

By the «operator of multiplying by the argument» in $L_\sigma^{(2)}$, we mean the operator defined on all those $f(t) \in L_\sigma^{(2)}$ for which $tf(t) \in L_\sigma^{(2)}$ and transforming these $f(t)$ into $tf(t)$.

Theorem 4. *If the set* $\mathfrak{R}_u$ *is dense in* $\mathfrak{H}_u$, *then every canonic function* $\sigma \in V_u$ *possesses almost everywhere a derivative equal to zero.*

2. There are cases where $\mathfrak{R}_u = \mathfrak{H}_u$, *i. e.* where every function $f(z) \in \mathfrak{H}_u$ is holomorphic on the real axis. In these cases the contents of Theorems 3, 4 admits of certain simplifications and sharpenings which we shall indicate below [cf. n° 5 of the present note and also our following note (2)].

3. We shall call a point a a regular point of the operator A under consideration, if the set $\mathfrak{M}_a = (A - aE)\mathfrak{D}_A$ ($\mathfrak{D}_A$ is the domain of definition of the operator A) is closed. Thus, every non-real point a is a regular point of the operator A.

It is easily shown that if a real point a is regular, then its sufficiently small neighbourhood contains only a finite number of points of the spectrum of every self-adjoint extension $\tilde{A}$ of the operator A; conversely, if this condition is satisfied at least for one of the self-adjoint extensions $\tilde{A}$, the point a is regular. Hence it follows that a sufficiently small neighbourhood of a regular point consists only of regular points.

There is no difficulty also in proving that if a certain real interval wholly consists of regular points, then the points of the spectra (*i. e.* the proper values) of two different self-adjoint extensions $\tilde{A}$ of the operator A situated in this interval are mutually separated.

Besides, for any point a of this interval there exists one and only one self-adjoint extension $\tilde{A}$ possessing in this point a proper value. Observe in passing that under the assumptions made with respect to the operator A (*viz.* that A is a simple operator with deficiency indices equal to one) every proper value of any self-adjoint extension $\tilde{A}$ of the operator A must be simple.

Theorem 5. *In order that a point* a *be a regular point of the operator* A, *it is necessary and sufficient that there should exist such a neighbourhood of the* a *and such an integer* $p \geqslant 0$ *that every function of the form* $(z-a)^p f(z)$, *where* $f(z) \in \mathfrak{H}_u$, *is holomorphic in this neighbourhood. This condition is satisfied, if it holds when* $f(z)$ *runs through a certain sequence* $\{f_n(z)\} \subset \mathfrak{H}_u$, *the linear envelope of which is dense in* $\mathfrak{H}_u$.

From Theorem 5 it easily follows that if a is a regular point of the operator A, then for a suitable choice of the scale u all functions $f(z) \in \mathfrak{H}_u$ will be holomorphic in one and the same neighbourhood of the point a; and conversely, if the scale u may be chosen in such a way that the indicated condition is satisfied, then a is a regular point of the operator A.

The second part of Theorem 5 readily follows from the following proposition.

Theorem 6. *The set* $\mathfrak{H}_u$ *possesses the property that for every complex* a *and positive* r *there exists a constant* $C = C(a, r)$ *such that if a function* $f_u(z) \in \mathfrak{H}_u$ *is holomorphic in the circle* $|z-a| < r$, *then*

$$|f_u(z)| \leqslant C\|f\| \quad \text{for } |z-a| < \tfrac{1}{2}r \qquad (\|f\| = (f,f)^{1/2})$$

4. We shall call a point a an u-regular point of the operator A, if there exists such a neighbourhood of the point a in which all functions $f(z) \in \mathfrak{H}_u$ are holomorphic.

From Theorem 5 it may be easily deduced that an u-regular point is such a regular point a of the operator A, for which $u \in \mathfrak{M}_a$.

Starting from any complete orthonormal system $\{\varphi_k\}$ in $\mathfrak{H}$ let us construct, as has been shown in n°4 of our note [1], the sequences of functions $\{\varphi_k(z)\}$ and $\{\psi_k(z)\}$. Then from Theorem 6 we deduce that a point a will be u-regular if and only if there exists such a neighbourhood of the point a in which all functions $\varphi_k(z)$ $(k=1, 2, \ldots)$ are simultaneously holomorphic. It turns out that in a regular point all functions $\psi_k(z)$ $(k=1, 2, \ldots)$ are holomorphic as well, and we have the following

Theorem 7. *The infinite series*

$$\sum_1^\infty |\varphi_k(z)|^2, \quad \sum_1^\infty |\psi_k(z)|^2$$

are uniformly convergent on any bounded closed set of u-regular points.

From Theorem 6 it follows also that if z is a fixed u-regular point of the operator A, then $f_u(z)$ $(f \in \mathfrak{H})$ is a linear functional in $\mathfrak{H}$, the norm of which is given by the formula

$$\sup_{f \in \mathfrak{H}_u} \frac{|f_u(z)|}{\|f\|} = \Big(\sum_1^\infty |\varphi_k(z)|^2\Big)^{1/2} = \frac{1}{d(u, \mathfrak{M}_z)}$$

where $d(u, \mathfrak{M}_z)$ is the distance of the element u from $\mathfrak{M}_z$.

Put

$$D(z) = \sum_1^\infty |\varphi_k(z)|^2$$

5. We shall call the operator A regular. if any point of the complex plane is a regular point for it.

Comparatively easily may be proved the following

Theorem 8. *The following assertions are equivalent:*

a) *The operator A is regular.*

b) *One of the self-adjoint extensions $\widetilde{A}$ of the operator A (and then also each of the others except one) has the inverse which is a continuous operator.*

c) *For a suitable choice of the scale u all functions $f(z) \in \mathfrak{H}_u$ are holomorphic on the real axis.*

If for a regular operator the scale u is chosen in such a way as was indicated in c), then we shall call it a regular scale. It happens that the regular scale u may be always chosen in such a way that the poles of any of the functions $f(z) \in \mathfrak{H}_u$ lie in the upper half-plane. For another choice of the regular scale we may achieve that $\mathfrak{H}_u$ will be of the real type, *i. e.* will possess the property that if $f(z) \in \mathfrak{H}_u$, then also $\overline{f}(z) \in \mathfrak{H}_u$.

If A is a regular operator and u a regular scale, then $\mathfrak{H}_u = \mathfrak{R}_u \subset L_\sigma^{(2)}$ for any $\sigma \in V_u$. By Theorem 3 the function $\sigma \in V_u$ will be canonical if and only if $\mathfrak{H}_u = L_\sigma^{(2)}$, *i. e.* if every function from $L_\sigma^{(2)}$ coincides almost everywhere (in the sense of the σ-measure) with a certain function from $\mathfrak{H}_u$. On the other hand, from Theorem 8 and the general considerations at the beginning of n°3 it follows that in the case under consideration every canonical function $\sigma \in V_u$ is a pure step-function, whose points of discontinuity accumulate only

at infinity; hereby the points of discontinuity of two different canonical functions $\sigma \in V_u$ are mutually separated, and the points of discontinuity of all possible canonical functions fill in the whole real axis.

Thus, the equality $\mathfrak{H}_u = L^{(2)}_\sigma$ characterizing a canonical function $\sigma \in V_u$ means that for every function $f(t) \in L^{(2)}_\sigma$ there is a function from $\mathfrak{H}_u$ (depending in general on σ) which coincides with the function $f(t)$ at all points of discontinuity of the canonical function $\sigma \in V_u$.

Theorem 9. *If A is a regular operator and u a regular scale, then in order that a function $f(z)$ meromorphic in the plane z should belong to the set $\mathfrak{H}_u$, it is necessary and sufficient that for a certain value of the constant C should be everywhere satisfied the inequality*

$$|f(z)|^2 \leqslant CD(z)$$

and that for at least one canonical function $\sigma \in V_u$ should be satisfied the condition

$$\int_{-\infty}^{\infty} |f(t)|^2 \, d\sigma(t) < \infty$$

The last condition will be evidently satisfied, if the function $f(z)$ is bounded on the real axis.

Kuibyshev Industrial Institute.

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MATHEMATICS

ON THE POSSIBILITY TO AVOID RIEMANN'S EXTENDED HYPOTHESIS IN THE INVESTIGATION OF PRIMES IN PROGRESSIONS

By U. V. LINNIK

(Communicated by I. M. Vinogradow, Member of the Academy, 9. XII. 1943)

Ever since the law of distribution of primes in progressions with fixed differences was proved (by J. Hadamard and Ch. de la Vallé-Poussin in 1896), the problems concerning the distribution of primes in the segments of progressions with variable differences $D \to \infty$, in particular, the classical problem on the least prime $p_{\min}(l, D)$ in a progression $Dx+l$, have been connected with Riemann's extended hypothesis (R*). This hypothesis runs as follows:

«If χ ranges through all the characters (mod D), then all the complex zeros of the series $L(s, \chi)$ lie on the straight line $\sigma = \frac{1}{2}$».

While studying the application of almost periodic functions to the theory of characters ([1–3]), the author revealed the interesting fact that the above hypothesis is not essential for the «qualitative» solution of the problem on $p_{\min}(l, D)$, as well as of other problems usually connected with the (R*). We may also dispense with nearly all information on the configuration of zeros of the L-series in the critical stripe except the well-known facts ([4]). The rôle of the hypothesis (R*) is thus reduced to improving the estimations in the results obtained without change of their «qualitative character». But we do need some information, however, on other points. Namely, it suffices to know that there is no «condensation» of zeros of every $L(s, \chi)$ in a certain rectangle.

Putting it in a precise form, we say that the series $L(s, \chi)$ has the property (d) (density property) if

Given any two closed circles of radius $1/\ln D$ *with centres within the rectangle* $0.9 \leqslant \sigma \leqslant 1$; $|t| \leqslant \ln^2 D$, *the ratio of the numbers of zeros (with their multiplicities) contained in these circles does not exceed an absolute constant* A.

In particular, for the problem of $p_{\min}(l, D)$ we have: if all the series $L(s, \chi)$ for the modulus D have the property (d) (a fixed number of exceptions is admissible), then

$$\sum_{\substack{n=1 \\ n \equiv l \pmod D}}^{\infty} \Lambda(n)\, e^{-n/N} = \frac{N}{\varphi(D)} \left\{ 1 + \vartheta \exp\left(-a_0 \frac{\ln N}{\ln D} \right) - \right.$$

$$\left. - \zeta(l) \exp\left(\beta(D) - 1\right) \frac{\ln N}{\ln D} + \frac{\vartheta'}{\ln D} \right\} \qquad (1)$$

where $|\theta| \leqslant 1$; $a_0 = a_0(A)$ is a constant dependent on A only; $\zeta(l)$ is a certain real character; $\beta(D) < 1$ is a function of D depending on the number of classes of ideals and of the fundamental unit in the fields $k(\sqrt{-D})$ and $k(\sqrt{D})$; D_1/D. Hence, in particular, follows the well-known hypothesis

$$\overline{\lim} \frac{\ln p_{\min}(l, D)}{\ln D} < \infty \text{ *} \tag{2}$$

for all progressions $Dx+l$ with $\zeta(l) = -1$**.

We shall outline the proof. For $\delta \in (0,1)$ we have (3)

$$\sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-\delta n} = \sum_{\rho_k \in \gamma} m_k \Gamma(\rho_k) \delta^{-\rho_k} + O(\ln^2 D) \tag{3}$$

where γ is the critical stripe: $\rho_k = \beta_k + it_k$ are zeros of $L(s, \chi)$; m_k, the multiplicity of zeros. For the sake of simplicity we shall suppose that, whatever χ may be, $\max \beta_k$ for $|t| \leqslant \ln^3 D$ is attained on the segment $|t| \leqslant 1$ (otherwise there arises an unessential complication). Let $\rho_\chi = \beta_\chi + it_\chi$ be one of the zeros with the maximal β_χ.

We have

$$\sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-\delta n} = e^{\beta_\chi x} \sum_k \Gamma_k \exp(-\sigma_k + it_k) x + O(\ln^2 D)$$

where $x = -\ln \delta$; $\Gamma_k = m_k \Gamma(\rho_k)$; $\sigma_k = \beta_\chi - \beta_k$. In other words:

$$\Phi(x, \chi) = \sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-\delta n} + e^{\beta_\chi x} \psi(x) + O(\ln^2 D) \tag{4}$$

where $\psi(x) = \sum_k \Gamma_k \exp(-\sigma_k + it_k) x$.

Put

$$\psi_1(x) = \psi(x) e^{-it_\chi x} = \sum_k \Gamma_k \exp[-\sigma_k + i(t_k - t_\chi)] x$$

Then, using the property (d) and integrating repeatedly (2), we establish the existence of such a_1, a_2, a_3 that

$$\int_{x_1}^{x_2} |\psi_1(x)|^2 dx = \int_{x_1}^{x_2} |\psi(x)|^2 dx > a_1 \ln D; \quad a_2 > 4$$

where $x_1, x_2 \in [a_2 \ln D, a_3 \ln D]$ (a_i depend on A only).

It is for this inequality only that the property (d) is required. Together with (4) this implies:

$$\int_{x_1}^{x_2} |\Phi(x, \chi) \exp(-\beta_\chi x)|^2 dx > a_5 \ln D \tag{5}$$

---

* In the subsequent publication the author will be able to prove (2) for all l with $(l, D) = 1$ without having recourse to any hypotheses.

** The case $\zeta(l) = +1$ needs special investigations. It is connected with the position of the single real zero of a single real series (mod D).

Let Q_{β_0} be the number of such series $L(s, \chi_1), L(s, \chi_2), \ldots, L(s, \chi_{Q_{\beta_0}})$ that have zeros for $\sigma \geqslant \beta_0$, $|t| \leqslant \ln^3 D$. Then for some $X_1, X_2 \in [a_6 \ln D, a_7 \ln D]$ (between which all x_1, x_2 lie) we shall have

$$\int_{X_1}^{X_2} \sum_{r=1}^{Q_{\beta_0}} |\Phi(x, \chi_r) \exp(-\beta_{\chi_r} x)|^2 \, dx \geqslant a_5 \ln D Q_{\beta_0} \tag{6}$$

The left-hand side will increase if we replace $e^{\beta_{\chi_r} x}$ by $e^{\beta_0 x}$ and extend the sum over all the characters χ. Hence

$$\int_{X_1}^{X_2} \frac{\sum_{\chi} |\Phi(x, \chi)|^2}{e^{2\beta_0 x}} \, dx \geqslant a_5 \ln D Q_{\beta_0} \tag{7}$$

We have further $\sum_{\chi} |\Phi(x, \chi)|^2 \ll \varphi(D) \sum_{n} \Lambda(n) e^{-\delta n} \sum_{n_1 \equiv n \pmod D} \Lambda(n_1) e^{-\delta n_1}$.

But, according to V. Brun ([4], [5]), we have for $\delta < 1/D^4$

$$\sum_{n_1 \equiv n \pmod D} \Lambda(n_1) e^{-\delta n_1} \ll \frac{\delta^{-1}}{\varphi(D)}$$

so that

$$\sum_{\chi} |\Phi(x, \chi)|^2 \ll \delta^{-2} = e^{2x} \text{ for } x \geqslant 4 \ln D.$$

Substituting this into (7), we obtain

$$a_5 \ln D Q_{\beta_0} \ll \int_{X_1}^{X_2} \frac{e^{2x}}{e^{2\beta_0 x}} \, dx = \int_{X_1}^{X_2} e^{2(1-\beta_0)x} \, dx \ll$$

$$\ll \frac{e^{2X_2(1-\beta_0)}}{1-\beta_0} \ll \frac{\exp(a_8(1-\beta_0)\ln D)}{1-\beta_0}$$

whence

$$Q_{\beta_0} \ll \frac{\exp(a_9(1-\beta_0)\ln D)}{(1-\beta_0)\ln D} \tag{8}$$

(the exceptional real series, if it is present, must be excluded from the above Q_{β_0} series). This is our fundamental estimation.

We further apply the formula:

$$\varphi(D) \sum_{n=l \pmod D} \Lambda(n) e^{-\delta n} = \sum_{\chi} \bar{\chi}(l) \sum_{n=1}^{\infty} \chi(n) \Lambda(n) e^{-\delta n} =$$

$$= \delta^{-1} - \sum_{\chi} \bar{\chi}(l) \sum_{\text{with respect to the zeros of } L(s,\chi)} \Gamma(\rho_k) \delta^{-\rho_k} + O(\delta^{-1}/\ln D) =$$

$$= \delta^{-1} + R + O(\delta^{-1}/\ln D)$$

To investigate the quantity

$$R = \sum_{\chi} \bar{\chi}(l) \sum_{\text{with respect to the zeros of } L(s,\chi)} \Gamma(\rho_k) \delta^{-\rho_k} \text{ for } \delta \leqslant 1/D^{a_0}$$

we divide the rectangle $0.9 \leqslant \sigma \leqslant 1$; $|t| \leqslant \ln^3 D$ into narrow stripes by the straight lines $\sigma = 1 - (m/\ln D)$; $m = 1, 2, \ldots, [0.9 \ln D + 1]$.

Taking into account that the number of zeros of every series $L(s, \chi)$ between t and $t+1$ is $O\{\ln[D(|t|+1)]\}$ and using the properties of the Γ-function, we easily obtain from the fundamental estimation (8) that the sum of the absolute magnitudes of the terms in R, for which $\beta_k \leqslant 1 - \frac{\ln \ln D}{\ln D}$, is $O(\delta^{-1} \ln D)$ for $\delta < 1/D^{a_0}$.

As to the terms for which $\beta_k > 1 - \frac{\ln \ln D}{\ln D}$, the problem is solved by means of the «fundamental lemma» of our paper (3): the distribution of zeros of $L(s, \chi)$ in critical stripe for $D \to \infty$ is such that

$$\sum_{\rho_k} \frac{m_k}{(\rho_k)^2} \exp(-\sigma_k C_0 \ln D) \ll 1$$

where $\sigma_k = 1 - \beta_k$ and m_k are multiplicities of the roots. The application of all these estimations leads us to (1) and (2).

We thus see that the property (d) enables us to dispense with Riemmann's hypothesis in this problem. The same is true in many other problems. A more detailed analysis shows that the property (d) may be replaced by a weaker condition.

Note that in our paper (3) the property (d) is established for the circles with centres on the line $\sigma = 1$.

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HYDRODYNAMICS

STABILITY OF TANGENTIAL DISCONTINUITIES IN COMPRESSIBLE FLUID

By L. LANDAU

(Communicated by P. L. Kapitza, Member of the Academy, 26. I. 1944)

The equations of hydrodynamics are known to admit in principle the existence of two types of discontinuity surfaces — shock waves and the so-called tangential discontinuities. In the latter the velocity component tangent to the surface (and, generally speaking, also the temperature) suffers a discontinuous change under continuous pressure. It is known also that in incompressible fluid tangential discontinuities are unstable with respect to infinitesimal disturbances and cannot therefore occur in reality. The object of the present paper is to prove that if the jump of velocity in discontinuity is large enough, so that the fluid may be looked upon as compressible, the tangential discontinuities are stable with respect to infinitesimal disturbances.

Let us consider a plane surface of a tangential discontinuity (we take this plane for the xy plane). By $\vec{v}_1$ and $\vec{v}_2$ we shall, respectively, denote the velocities of the fluid to the one and the other side of the discontinuity, tangent to its plane. One of them may be put at zero without impairing the generality of our reasoning. Indeed, it can always be brought to this value by a suitable choice of the coordinate system.

Let us, for instance, put $\vec{v}_1 = 0$, and denote $\vec{v}_2$ simply by $\vec{v}$; the direction of $\vec{v}$ is defined to be that of the x axis.

Following the ordinary method of stability determination with respect to infinitesimal disturbances, we suppose the basic motion to by superposed by a disturbance in which all quantities are related to the time and to coordinate x through the factor $e^{-i\omega t + ikx}$. Let us examine our fluid from that side of the discontinuity surface ($z > 0$) on which $\vec{v}_1 = 0$. For the disturbance of pressure p_1' (just as for any small disturbance in incompressible fluid) we have the wave equation

$$\Delta p_1' - \frac{1}{c_1^2}\frac{\partial^2 p_1'}{\partial t^2} = 0$$

where c_1 is the velocity of the sound in fluid 1 (liquid). We look for p_1' in the form

$$p_1' = \text{const} \cdot e^{-i\omega t + ikx + i\varkappa_1 z} \qquad (1)$$

(If $\varkappa_1$ is a complex number, it has to be so chosen that its imaginary part be positive). The wave equation leads to the relation

$$\frac{\omega^2}{c_1^2} = k^2 + \varkappa_1^2 \qquad (2)$$

which links up ω, k and $\varkappa_1$. Further, for the disturbance of velocity v'_{1z} we have, in accordance with the equation of Euler (in which terms of higher orders are omitted):

$$\frac{\partial v'_{1z}}{\partial t} = -\frac{1}{\rho_1}\frac{\partial p'_1}{\partial z}$$

Together with (1) it gives

$$v'_{1z} = \frac{\varkappa_1 p'_1}{\omega \rho_1} \tag{3}$$

Let $\zeta = \zeta(x, t)$ be the displacement of the discontinuity surface points along the z axis, due to disturbance. We have obviously $\frac{\partial \zeta}{\partial t} = v'_{1z}|_{z=0}$. If ζ is sought in the form $\zeta = \text{const} \cdot e^{-i\omega t + ikx}$, we shall find $v_{1z} = -i\omega\zeta$ or, after substitution of (3):

$$p'_1 = -\frac{i\omega^2 \rho_1 \zeta}{\varkappa_1} \tag{4}$$

In fluid 2 (gas) we look for p'_2 in the form of $p'_2 = \text{const} \cdot e^{-i\omega t + ikx - i\varkappa_2 z}$ where the imaginary part of $\varkappa_2$ is again positive; the relation between ω, k and $\varkappa_2$ cannot be found directly from the wave equation, because the latter is not valid unless the medium is immovable as a whole. If, however, we pass from the x, y, z system to $x' = x - vt$, $y' = y$, $z' = z$, we shall find that in this new system of coordinates the fluid 2 is at rest, and so the wave equation will be applicable.

For p'_2 in coordinates we have $p'_1 = \text{const}\ e^{-i\omega t + ikvt}\, e^{ikx' - i\varkappa_2 z'}$ and the wave equation yields

$$\frac{(\omega - kv)^2}{c_2^2} = k^2 + \varkappa_2^2 \tag{5}$$

Instead of (4) we have here

$$p'_2 = \zeta \frac{i(\omega - kv)^2 \rho_2}{\varkappa_2} \tag{6}$$

The pressure on the discontinuity surface must be continuous, *i. e.* $p'_1 = p'_2$. Substituting (4) and (5) we obtain

$$\frac{\rho_1 \omega^2}{\varkappa_1} + \frac{\rho_2 (\omega - kv)^2}{\varkappa_2} = 0 \tag{7}$$

After elimination of $\varkappa_1$, $\varkappa_2$ from equations (2), (5) and (7) we obtain the following equation which defines «the frequencies» ω:

$$\frac{1}{c_1^2 \rho_1^2} = \frac{\frac{\omega^2}{k^2} - c_1^2}{\frac{\omega^4}{k^4}} = \frac{1}{c_2^2 \rho_2^2} \frac{\left(\frac{\omega}{k} - v\right)^4 - c_2^2}{\left(\frac{\omega}{k} - v\right)^4} \tag{8}$$

This is an equation of the 6th degree which for sufficiently high v has six real roots.

The truth of this may be easily verified by means of simple geometrical construction. Note that the roots of equation (8) are intersection points of the curve

$$\eta = \frac{1}{c_1^2 \rho_1^2} \frac{\xi^2 - c_1^2}{\xi^4}$$

with another curve of the same type shifted along the axis ξ to a distance v, relative to the former curve. It will be obvious that there always exists such a value v_k of the velocity v that for $v > v_k$ all frequencies ω are real, so that the basic motion is stable.

The value v_k may easily be calculated for a discontinuity in compressible fluid which may be considered an ideal gas having its specific heat independent of temperature. For an ideal gas $c^2 = \gamma \frac{p}{\rho}$; as the pressure to either side of the

discontinuity is equal, we have $c_1^2\rho_1 = c_2^2\rho_2$ ($\gamma = c_p / c_v$ is the specific heat ratio of the gas; it is supposed that the same gas is on either side of the discontinuity, or else there are two different gases, but with an equal value of γ). Accordingly, we rewrite (8) thus:

$$\frac{c_1^2}{\omega^2} - \frac{c_2^2}{(\omega - kv)^2} = k^2\left(\frac{c_1^4}{\omega^4} - \frac{c_2^4}{(\omega - kv)^4}\right)$$

Reducing by the factor $\frac{c_1^2}{\omega^2} - \frac{c_2^2}{(\omega - kv)^2}$ (and thus discarding the two roots of the equation that are known beforehand to be real) we get

$$\frac{1}{c_1^2 c_2^2 k^2} = \frac{1}{c_1^2(\omega - kv)^2} + \frac{1}{c_2^2\omega^2} \qquad (9)$$

This equation is of degree four, and has always two real roots, while its other two roots are not real unless v exceeds a certain value v_k; for $v = v_k$ we have a multiple root. The value v_k can be obtained by eliminating ω from equation (9) and from the equation obtained by differentiating (9) with respect to ω.

Finally we obtain

$$v_k^{2/3} = c_1^{2/3} + c_2^{2/3} \qquad (10)$$

If we have the same temperature on either side of the discontinuity, then $c_1 = c_2 = c$ and $v_k = 2\sqrt{2}\,c$.

The value v_k can also be computed for the case where we have gas to one side of the discontinuity and liquid, to the other. Supposing that the density of the liquid ρ_l is large compared to the density of the gas ρ_g we get

$$v_k = \frac{2\rho_l c_l^2}{\rho_g c_g}.$$

This is, however, a very large value, and in practical cases it is never attained.

The following general remark needs to be made with regard to the results obtained. We have only proved the stability of a discontinuity (when there is a sufficiently large jump of velocity) in respect to infinitesimal disturbances. Yet in principle it may happen that the discontinuity will remain unstable in respect to disturbances of finite amplitude (as is the case, *e. g.*, with Poiseuille's flow). This would imply a practical instability of tangential discontinuity in the case of large jumps of velocity also. This appears to be testified by the experiments carried out by J. B. Zeldovich and J. B. Chariton who have kindly made available their results to the author. In their experiments they created a jet of hydrogen moving at a rate greater than that set up by formula (10). They expected such a jet, if the corresponding tangential discontinuity should be stable, to be sharply separated from the surrounding air, in contrast with the ordinary picture of turbulent mixing. This they failed to observe, however.

Institute of Physical Problems.
Academy of Sciences of the USSR.

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PHYSICS

IONIZATION BURSTS (HOFFMANN STÖSSE) AND AUGER SHOWERS AS RELATED TO ALTITUDE

By D. W. SKOBELZYN, Corresponding Member of the Academy

In an earlier paper (1) I have given a formula that establishes a very simple relation between the records of atmospheric (Auger) showers as taken by the coincidence method or by observing the ionization bursts produced by these showers.

The just mentioned expression for the frequency of bursts and the formula for the number of impulses recorded by the coincidence circuit involve the same integrals of certain functions of the energy of primary particles. This fact had already been pointed out earlier.

On account of it the correspondence between the number of coincidences and the number of bursts per unit time would be perfectly univalued and independent of the assumptions concerning the primary spectra, if the limits of integration over the energies of the primary particles, or, in other words, the mean effective energies, were equal in both cases.

However, the latter condition is not fulfilled, *i. e.* the region of effective energies, and the boundaries of integration with respect to primary spectrum energies prove to be somewhat different.

Accordingly, the altitude dependence will not perhaps be the same for showers as for bursts.

As the calculations referred to below will show, the altitude rate of bursts is rather slow in growing compared to the increase in the number of double or triple coincidences. Contrary to this, the available experimental data lead to the conclusion that the number of bursts observed in thin-walled chambers grows with altitude much more rapidly than does the number of Auger showers recorded by the method of double coincidences.

Therefore if at a certain height (*e.g.* at sea level) a correspondence between the number of bursts per unit time and the number of Auger showers observed by the method of coincidences does take place, one will find nevertheless that at high altitudes (4360 m) there is a notable divergence between them.

The consequences of this will be discussed in the next paper. Here we shall dwell on those quite elementary computations that permit the probability of recording bursts and its dependence on the height of the observation point to be determined.

We shall proceed on the following assumptions:

1. The spectrum of primary energies is given by the relation

$$f(E)=\frac{A}{E^{\gamma}}, \qquad \Phi(E)=\frac{1}{\gamma}\frac{A}{E^{\gamma}}, \qquad \gamma=1.8 \tag{1}$$

[for the meaning of symbols see (1)].

2. In accordance with the cascade theory ([3]) the density ρ of the space distribution of particles in the shower is related to the radius r as follows:

$$\rho \sim 1/r^{2-s} \text{ for } r < R \tag{2}$$

where R is the radius of the shower; s, the well-known parameter of the cascade theory introduced by Landau and Rumer; $R = 7 \cdot 10^3$ cm at a height of about 3500 m, which is in agreement with the Auger curve for the relation of the number of coincidences to the distance between the counters ([2]).

3. For simplicity sake put

$$\rho = 0 \text{ for } r > R \tag{3}$$

The number of the ionization bursts exceeding a certain fixed value i_0 may be expressed as follows

$$\nu = \int_0^R 2\pi \Phi(E) r \, dr \tag{4}$$

or

$$\nu = \int_{E_{min}}^{E_{max}} \pi r^2 f(E) \, dE \tag{5}$$

If to a given minimum value of the burst size there corresponds a density $\rho = i_0/\sigma$ (σ, cross-section area of the chamber), or, in other words, if we consider showers of high particle density exceeding the given value of ρ, then, in accordance with (2) E in the integrand of (4) and (5) is linked with r by the following relation

$$N(E) = \frac{2\pi R^s r^{2-s} \rho}{s} \tag{6}$$

where $N(E)$ is the number of particles in a cascade produced by a primary particle with energy E; $\rho(r) = \rho_0/r^{2-s}$, where

$$\rho_0 = \frac{sN}{2\pi R^s} \tag{7}$$

according to the normalizing condition $2\pi \int_0^R \frac{\rho_0 dr}{1-s} = N$.

In the case discussed here E_{min} in formulae (4) and (5) can to all practical purposes be put at zero.

As for the upper limit of integration, E_{max}, there are two possible cases.

I.

$$N(E_R) = \frac{2\pi R^2 \rho}{s} < N_{max} \tag{8}$$

where N_{max} is the highest possible number of particles in a cascade shower at a given altitude.

Condition (8) means that with E sufficiently large, the chamber will record a burst when the axis of the shower is at any distance r up to $r = R$.

This is exactly the case within the range of altitude to which the observations have so far been confined.

II. i_0 exceeds the value of the burst size at the edge of the shower, which corresponds to the maximum number of particles in a shower (at a given altitude). This may only occur at a very high elevation or else when $\rho = i_0/\sigma$ is very large.

$$N_{max} < N(E_R) \tag{9}$$

It will then be obvious that E_{max} is that value of the primary energy above which $N(E)$ practically attains the highest possible value (for a given altitude).

The maximum distance from the axis of the shower at which the latter may produce in the chamber a burst of required size is

$$r_{max} = R \text{ in the case I}$$

$$r_{max} = \left[\frac{s N_{max}}{2\pi R^2 \rho}\right]^{\frac{1}{2-s}} \text{ in the case II.}$$

The conditions adopted below are those corresponding to case I.

The integrand in (4) gives the distribution along the radius r of the bursts observed in the chamber (with sizes exceeding a given value) (Fig. 2).

The integrand in (5) gives the distribution of the observed bursts according to the energy of the primary particles (Fig. 1).

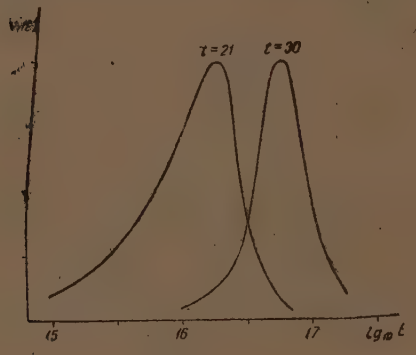

Fig. 1

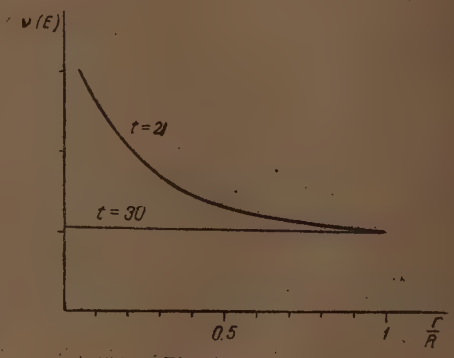

Fig. 2

Making use of the formulae of the cascade theory we may suppose that

$$N(E) \sim E^s, \tag{10}$$

where s is a parameter varying with E and with the depth t of the given layer relative to the boundary of the atmosphere.

Denoting by $\nu(E)\frac{dE}{E}$ the number of bursts due to primary particles having energies ranging within the interval dE, and by $\nu(r)\,dr$ the number of bursts due to showers whose axis is at a distance r to $r+dr$ from the chamber (irrespective of their energy), and putting

$$\nu(E) = B\,\nu_1(E) \tag{11}$$

where

$$B = A\pi R^2 E_R^{-\gamma} \tag{12}$$

we have

$$\nu_1(E) = (E/E_R)^{\frac{\gamma s}{2-s}-\gamma} \text{ for } E < E_R \tag{13}$$

and

$$\nu_1(E) = (E/E_R)^{-\gamma} \text{ for } E > E_R \tag{14}$$

$$E_R = k\left(\frac{2\pi R^2 \rho}{s}\right)^{1/s} e^{-\lambda(s)\,t} \tag{15}$$

k is a factor slightly varying with S and t; $\lambda(s) < 0$; the symbol $\lambda(s)$ corresponds to the notation of Rossi-Greisen (4).

Figs. 1 and 2 give the corresponding curves in relative units for different elevations: $\nu(r) \sim \Phi(E) r$ [$\nu(r) = d\nu/dr$].

The numerical value of ρ is selected in accordance with the observations made by Montgomery (5); $\rho = 5 \cdot 10^2 M^{-2}$; s for a given t corresponds to E_R.

Fig. 2 gives the curves $\nu_1(E)$ for some fixed altitudes.

The areas of these curves $\sum$* are tabulated below.

t	$\lg_{10} E_R$	s	$\sum$	ν	C_2
30	16.8	1.3	1.1	1	1
26	16.5	1.2	1.4	5.5	5.8
21	16.3	1.1	2.6	13	32
18	16.3	1	4	16	45

Miltiplying $\sum$ by (12) we find ν, whose values are given in the fifth column of the table in relative units**.

In the sixth column is given the number of double coincidences (1) (likewise in relative units and making allowance for the dependence of R upon the height of the observation point).

The variation of ν and C_2 with the altitude (columns 5 and 6) is brought out by curves in Fig. 3.

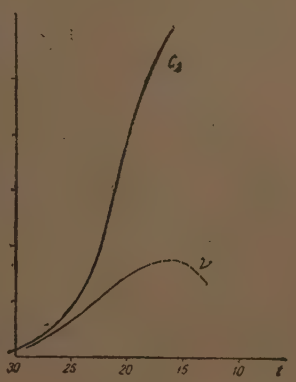

Fig. 3

These curves are seen to diverge markedly. It should be noted that even at not very high altitudes one may expect to find a maximum in the altitude curve of the bursts (due to Auger showers).

We have supposed for simplicity that the section of the shower is distinctly bounded by a circumference of radius R and that upon crossing this boundary the density ρ falls down to zero.

One may easily be convinced that the presence of a «tail» in the density distribution beyond the boundary $r = R$ (*e. g.* of the $\rho \sim e^{-r/R}$ type), though tending to diminish*** the computed values, cannot materially affect the dependence on the altitude; and if the altitude curve does undergo a slight change, it merely consists in its sloping more gently.

That the bursts observed in thin-walled chambers at sea level are due to Auger showers seems by now to have been established beyond discussion. Direct evidence in support of this statement is furnished by an array of experimental data.

Aside from the facts pointed out previously, one should note the existence of a characteristic sharply expressed transition effect**** observed on passing from

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* In calculating these areas one may use the relation (13) with s constant, if the band of effective energies is narrow enough, as is indeed the case at small altitudes.

** In computing ν and C_2 it has been taken into account that R varies inversely as the density of the medium in which the shower is formed. To make allowance for the influence of the density, the factor $(t/t_0)^{\frac{2\gamma}{s}-2}$, since $\nu \sim R^{2-2\frac{\gamma}{s}}$, has been introduced.

*** Since the presence of such a «tail» is equivalent to a certain increase of the effective radius and, accordingly, of the energy E_R.

**** The transition maximum observed in this case (Montgomery has reported a more than tenfold increase of the number of bursts) is caused by the concentration of shower particles connected with the decrease of the radius R of the shower. As formulae (11), (12) and (13) show, the increase of the number of bursts due to this cause is proportional to $R^{\frac{2\gamma}{s}-2}$.

the atmosphere to a dense medium with practically the same atomic number [see also (6)].

Formulae (11)—(15) permit the absolute number of bursts per unit time to be evaluated whenever the primary spectrum is given. If we proceed from the expression for the intensity distribution in the primary spectrum, which is in agreement with the results of observations of Auger showers by the method of coincidences, then computation according to the above scheme will yield values which accord well in the order of magnitude with the observations of different experimenters.

Thus, relative to the level $t = 26$, for which our former computations were made (1), and to bursts of the magnitude arbitrarily chosen by us, we obtain according to (5) and (11—15)

$$\nu = \frac{A\pi R^2 \Sigma}{E_R^\gamma} = \frac{7 \cdot 10^{20} \cdot \pi \cdot 5 \cdot 10^7 \cdot 1.4}{10^{15.5} \cdot 1.5} = 0.3 \text{ hour}^{-1}$$

The constant A is selected to correspond to the number of double coincidences C_2 observed at a fixed altitude (sea level).

The corresponding value of ν is, according to Montgomery (5), 0.4 hour^{-1}*. The same order of magnitude is obtained with Karmichael's chambers, and also from recent observations by Lapp (6).

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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* The relation of the number of double coincidences observed at a given level to the number of bursts of the magnitude discussed here depends upon the assumptions as to the distribution in the primary spectrum (since the effective energy regions are not the same in these two cases).

The choice of the function $f(E)$ of the primary spectrum affects the dependence on the altitude, and the distribution of bursts according to their magnitude. The relation (1) chosen by us gives in the main a correct altitude run of C_2 and corresponds also for very large ρ to the curve $\nu = \varphi(\rho)$. In the region of smaller ρ the experimental curve $\varphi(\rho)$ of the bursts observed at sea level has a far less gentle slope, which is not consistent with (1). The reason of this conflict is not clear.

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PHYSICS

ON THE POLARIZED LUMINESCENCE OF DYE-STUFFS IN RIGID SOLUTIONS

By P. P. FEOFILOV

(Communicated by S. I. Wawilow, Member of the Academy, 30. IV. 1944)

The introduction of organic dye-stuffs into rigid vitreous solutions not infrequently leads to the appearance of a bright afterglow which lasts for some seconds and is often identical in its spectral composition with the instantaneous fluorescence of the respective dye-stuffs. Besides, a longer-wave radiation is observed, mostly at low temperatures. To explain these phenomena one has to assume the existence of peculiar molecular states (metastable or phosphorescent states) possessing a considerable surplus of energy compared to the normal molecular state and, at the same time, rather stable ([1-3]).

Below the author gives some new data on the polarization properties of the luminescence of rigid solutions.

Comparing the polarization of the instantaneous fluorescence with that of the luminescence of long duration one arrives at certain conclusions important for the characteristic of the metastable states of organic molecules, as well as for the entire theory of polarized luminescence.

The objects under investigation were boric acid and sugar phosphors prepared in the ordinary way [see, for instance ([2,4,5])]. For the measurements transparent specimens were chosen free from stresses which could affect the obtained results. The observations were carried out at room temperatures; thus, all the data obtained refer to phosphorescence spectrally identical with instantaneous fluorescence.

1. Dipole Character of the Phosphorescence of Dye-stuffs

Using the polarization method proposed by S. I. Wawilow ([6]) we have determined the nature of elementary oscillators in the case of phosphorescence of an acridine orange dye-stuff in a rigid sugar solution. To avoid systematic errors that might possibly arise from inaccurate calibration of the polarimeter, the measurements of curves $p = f(\eta)_{\chi=\pi/2}$ were conducted in parallel for the phosphorescence of a rigid solution and for the instantaneous fluorescence of a glycerine-water solution of the same dye-stuff, the latter being known beforehand to be a dipole radiation. The measurements were made in a phosphoroscope of the Becquerel type; a total reflection prism located near the specimen allowed to set up the conditions $\chi = \pi/2$. While observing the fluorescence of liquid solution, the rotation of phosphoroscope was stopped and its discs set in such a position that the exciting light and the fluorescence could pass simultaneously (crossed filters were used for observation). The viscosity of the glycerine-water solution in the control experiments with fluorescence had such a value that the limiting pola-

rization of fluorescence was equal to that of the phosphorescence of a rigid solution (24.6 per cent).

As seen from Fig. 1, the experimental points for phosphorescence as well as for instantaneous fluorescence fall down the same curve (with some scattering due to the experimental error), *i. e.* the phosphorescence of rigid solutions as well as the short fluorescence of dye-stuffs corresponds to electric dipole radiation. However, this common curve is situated a little lower than that calculated by S. I. Wawilow for the case dipole→dipole.

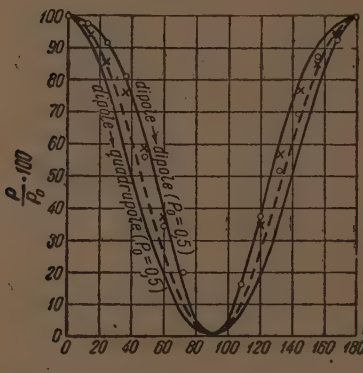

Fig. 1. Dye-stuff—acridine orange. ×, in glycerine-water solution; ⊙, in a sugar phosphor

It is easy to show that this is connected with the fact that the limiting polarization is, in our case, essentially lower than the calculated limiting value (0.5), and that when the limiting polarization is very small, some cases of multipoles become indiscernible.

Let us consider for this purpose the expressions obtained by S. I. Wawilow for relative intensities of two perpendicular components of radiation in some given direction [(6), formulae for the case $d \rightarrow d$ (12) and (13)]. For the particular case that interests us ($\chi = \pi/2$) they can be rewritten as

$$\sum z^2 = \frac{2\pi}{15}(1+2\cos^2\eta); \qquad \sum y^2 = \frac{2\pi}{15}$$

If the limiting polarization is less than 0.5, the radiation may be represented as a sum of two radiations: one, the anisotropy of which is described by formulae (12) and (13), and a second fully isotrope, having its relative intensity determined by the limiting polarization [$P(0)$]. In this generalized case the formulae become

$$\sum z^2 = \frac{2\pi}{15}(1+2\cos^2\eta)+\alpha; \qquad \sum y^2 = \frac{2\pi}{15}+\alpha$$

where α is some intensity which does not depend on η because the isotrope radiation has spheric symmetry. Hence

$$P(\eta) = \frac{\cos^2\eta}{1+\cos^2\eta+\frac{15\alpha}{2\pi}}$$

and, if we introduce $P_0 = P(0)$,

$$P(\eta)_{d\rightarrow d} = P_0\frac{\cos^2\eta}{1-P_0\sin^2\eta}$$

$P(\eta)_{\chi=\pi/2}$ for the case dipole → quadrupole may be calculated in a similar way

$$P(\eta)_{d\rightarrow q} = P_0\frac{\cos^2\eta}{1+P_0\cos^2\eta}$$

Thus, in the limiting case, where P_0 tends to 0, the cases $d\rightarrow d$ and $d\rightarrow q$ become indiscernible, because the curves $P(\eta)$ tend to a common curve $P = P_0\cos^2\eta$*.

As seen from Fig. 1, the measured polarization values are situated between the curves $d\rightarrow d$ when $P_0 = 0.5$ and $d\rightarrow d$ when $P_0 \rightarrow 0$.

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* It may be shown that the cases $q\rightarrow d$ and $q\rightarrow q$ become also indiscernible when P_0 is small. Thus, when $P_0\rightarrow 0$ the course of the curves $P(\eta)$ is determined only by the nature of the absorbing oscillators.

Our results refer to a phosphorescence spectrally identical with instantaneous fluorescence. S. Weissmann and D. Lipkin have shown recently (7) by means of wide-angle interference method that the longer-wave luminescence observed at low temperatures is also of dipole nature. Apparently, all the optical processes in organic dye-stuffs may be described by electric dipole oscillators.

2. Polarization Spectra of Luminescence of Rigid Solutions

Polarization spectra (dependence of polarization of luminescence on the wavelength of the exciting light) express the spectral course of the anisotropy of absorption (8) of luminescent molecule. Using the experimental arrangement described in a previous paper (8), we carried out a comparative study of polarization spectra of instantaneous fluorescence and phosphorescence. A special phosphoroscope of the Becquerel type placed before the exit slit of a monochromator permitted the phosphorescence to be separated from fluorescence. Observations without a phosphoroscope gave information concerning the entire luminescence, consisting mainly of fluorescence of short duration. In the majority of investigated cases the obtained curves coincide, *i. e.* the polarization spectra of phosphorescence and fluorescence prove to be identical (see, for instance, Fig. 2, *a*). In some cases (*e. g.* in the case of boric acid phosphor with trypaflavine, Fig. 2, *b*) a slight but unmistakable shift of spectra was observed.

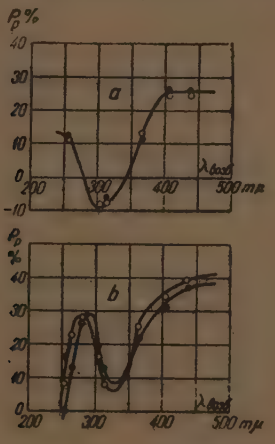

Fig. 2. *a*, fluorescein in boric phosphor; *b*, trypaflavine in boric phosphor; ●, light emission under continuous excitation; ○, phosphorescence

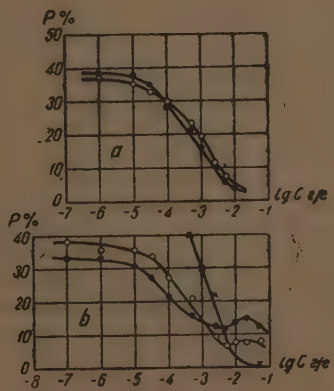

Fig. 3. *a*, eosin; *b*, trypaflavine; ●, light emission under continuous excitation; ○, phosphorescence; ×, relative yield

The coincidence of polarization spectra gives evidence of the identity of the anisotropy of «centres» responsible for both luminescences, and thus appears a convincing argument for the identity of these «centres».

The undoubtedly existing short-wave displacement of the polarization spectrum of fluorescence of trypaflavine in boric acid is possibly due to the unequal transition probability to the metastable (phosphorescent) state from different vibrational levels of the molecule. Indeed, when a molecule is excited in the region where two absorption bands overlap, higher vibrational levels of the longer-wave band and lower levels of the shorter-wave one will be excited. If the probability of transition to the phosphorescent level is greater in the case of strongly excited vibrations of the molecule (which agrees with the existing hypotheses on the nature

of phosphorescent state), the relative part of long-wave bands in phosphorescence will be greater, and the polarization spectrum of phosphorescence will be shifted to the short-wave side of the spectrum. It is possible that in this respect molecules responsible for phosphorescence and fluorescence somewhat differ from each other.

The coincidence of polarization spectra of phosphorescence and fluorescence is additional evidence that the characteristisc dependence of polarization on the wave-length of the exciting light is not due to the oscillator changing its orientation during the existence of the excited state. If kinetic factors played an important part in the formation of the polarization spectrum, there can be no doubt that the form of the latter would be affected by the enormous increase in the duration of the excited state.

3. Concentrational Depolarization (c. d.) of the Luminescence of Rigid Solutions*

The development of the theory of concentrational depolarization may be accomplished only up to some constant (the probability of depolarizing energy transfer) determined by experiment (°). To get a deeper insight into the mechanism of depolarizing transfers, one has to find out the basic regularities underlying this process—to clear up the conditions under which this process can take place.

The concentrational depolarization of luminescence is determined by the process in which the excited molecules are involved throughout the period of existence of the excited state. By shortening the duration of the excited state a considerable diminution of the c. d. can be obtained. This appears to be at variance with the existence of polarization of phosphorescence in rigid solutions. In so far indeed, as the duration of the excited state in these solutions is approximately 10^9 times as great as in the case of liquid solutions, the c. d. in them must develop at concentrations of orders 10^{-12} g/cm^3, which is far beyond the usual range of concentrations of the dye-stuff in these phosphors. In order to find out the reason of this inconsistency, we have carried out a comparative study of c. d. of phosphorescence and fluorescence (when observing the fluorescence of short duration there existed a certain admixture of phosphorescence). For this purpose sugar phosphors with different concentrations of the dye-stuff were prepared. The observations were carried out either in a phosphoroscope or by means of crossed light-filters. As seen from Fig. 3, the phosphorescence depolarization process develops at the same concentrations (10^{-3} g/g) as in the case of the depolarization of fluorescence. In the case of benzoflavine or eosine the course of the curves is quite identical, in the case of trypaflavine c. d. develops somewhat more rapidly than the phosphorescence depolarization. The curves thus obtained closely approach the respecting curves for liquid solutions. Thus, in the case of trypaflavine at concentrations of order $5 \cdot 10^{-3}$ g/g, just as in the case of liquid solutions, there is observed a minimum of polarization after which it somewhat increases again. The position of the minimum coincides, as it does in the case of glycerine solutions, with a steep fall of the luminescence efficiency curve which coincides in its turn with the concentrational quenching curve of the liquid solution. The last phenomenon—the coincidence of concentrational quenching(c.q.)curves in sugar phosphors and liquid solutions—was first discovered by W. L. Lewschin ([11]). We observed this coincidence for the curves of c. d. as well. This evidence seems to speak strongly for the common nature of the primary processes to which the two phenomena connected with the concentration increase are due.

The identity of concentrational curves for phosphorescence and fluorescence proves that the processes which determine c. d. and c. q. either do not take place while the molecule is in metastable (phosphorescent) state or proceed with negligible probability. Thus, the energy transfer between an unexcited molecule and a molecule in phosphorescent state is impossible. Only after passing to the

* The experiments described below were performed in collaboration with M. J. Golubchik, undergraduate of the Kiev State University.

labile state is it possible for the molecule to transfer its energy to the neighbouring molecules, the course of the phenomenon being in general the same as in liquid solutions. It seems very likely, assuming that the probability of energy transfer is here the same as in liquid solutions, that the duration of the labile excited state to which the molecule passes after the metastable period is as long as the duration of the excited state in liquid solutions.

The coincidence of concentrational curves for instantaneous fluorescence and phosphorescence, as well as the fact that these luminescences are identical in spectral composition and in their polarization properties shows the phosphorescent state to be a peculiar kind of inert state, which leaves no traces on the excited molecule. The impossibility of resonance energy transfer between unexcited and phosphorescent states of a molecule seems to be exceedingly important for understanding the energy transfer process.

The absence of depolarizing processes developing during the phosphorescent state of the molecule is proved also by our observations of polarization in different stages of the sugar phosphor decay. We succeeded only in the qualitative observations which showed that the polarization keeps its value throughout the process of phosphorescence decay ($\sim$1—2 sec.). If depolarizing transfers were possible while the molecule is in phosphorescent state, the degree of polarization would fall off with decay of luminescence as it does in the case of spontaneous fluorescence ([12]).

This work was carried out under the guidance of S. I. Wawilow, Member of the Academy, to whom I tender my sincere thanks.

Laboratory of Luminescence,
State Optical Institute.

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CHEMISTRY

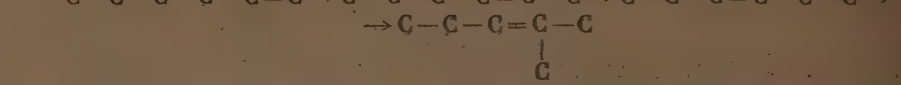
ON THE ISOMERIZATION OF HEXENE-1 INTO TETRAMETHYL-ETHYLENE

By M. A. CHELTZOVA and A. D. PETROV

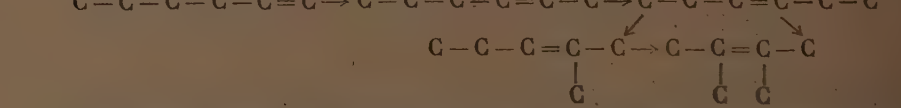
(Communicated by A. N. Nesmeyanov. Member of the Academy, 27. I. 1944)

Isomeric transformations, consisting in a shift either of the double bond or of existing side-chains, has been observed for a long time and in many cases (1). The formation of branched hydrocarbons starting from those having a normal chain was not brought about until 1936, when it was carried out simultaneously by Serebriakova, Rudkovsky and Frost (2) (isomerization of normal butene) and by the present authors (3) (isomerization of hexene-1 and a mixture of octene-1 and octene-2). At that time, however, we only stated the presence of branched hydrocarbons in the product of hydrogenation of the isomerized substance (through chlorination using a $SbCl_5$ solution) and studied the dependence of the yield of branched hydrocarbons on temperature, pressure and other factors, but left without attention the structure of the branched hexenes formed as a result of isomerization. Somewhat later Petrov and Schukin (4) were successful in converting hexene-1 into dimethyl-ethyl-ethylene using phosphoric acid as a catalyst.

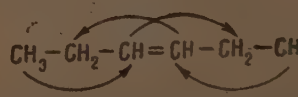
This reaction could be expressed by the following scheme:



In the present investigation, over a more active catalyst ($ZnCl_2$) in the presence of a hydrogen halide (HCl) used as a promoter, the authors were able to achieve further and more profound isomerization and to obtain from hexene-1 a hydrocarbon having two side-chains (tetramethyl-ethylene). Its formation proceeds either through dimethyl-ethyl-ethylene or is the result of direct isomerization of symm. diethyl-ethylene along the same scheme. Therefore the reaction can be expressed by the following equation:



With more details the mechanism of this reaction can be expressed by an intramolecular β-rearrangement of the hydrogen atoms and of the methyl radical



As is well known, A. E. Favorsky (5) explains through β-rearrangements of hydrogen atoms the isomeric transformations discovered by him in the series

of acetylenic and diethylenic hydrocarbons involving no change of the carbon skeleton. In this general scheme we make a further step, allowing for the isomerization involving a change in the carbon skeleton β-rearrangements both of hydrogen atoms and of radicals.

Experimental. Hexene-1 was prepared by the dehydration of *n*-hexyl alcohol over alumina at 350°. From 700 cm³ of alcohol 500 cm³ of hexene-1 were obtained, b. p. 62—64°; 18 cm³ of this product were hydrogenated with a platinum catalyst. The hydrocarbon obtained had the following properties: b. p. 66.5—68.5°; n^{20}_D =1.3757 (*n*-hexane has the following physical constants: b. p. 68.8°, n^{20}_D =1.37506) (⁶).

The content of branched isomers in this hexane determined according to Schaarschmidt-Moldavsky was below 1 per cent.

Experiments on the isomerization of hexene-1 were carried out under pressure. The initial pressure of nitrogen was 50 atm. As a catalyst $ZnCl_2$ deposited on pumice was used, the ratio being 1 : 1. In every run 100 cm³ of the hexene were taken. Conditions of runs and results obtained are given in the table.

Catalyst	Duration of heating in hours	Temperature in °C	B. p. of the isomerized product	Yield of fraction boiling below 80° in cm³	Yield of fraction b. p. 70—76° in cm³	Results of bromination
35 g $ZnCl_2$ on pumice 1:1	2	375	55—80	45	4	No crystalline product
Do	6	»	47—80	39	7	Do
35 g $ZnCl_2$ on pumice 1:1+2% HCl	2	»	40—80	35	7	Crystalline products obtained
Do	4	»	32—80	32	10	Do
Do	6	»	27—80	29	12	»

As may be seen from the table, isomerization with the formation of tetramethyl-ethylene takes place only in those runs in which HCl is present.

The isomerized products obtained in the last 3 runs were united and subjected to distillation with a Vigreux column. After two distillations 22 cm³ were obtained of a fraction, b. p. 70—76°, the physical constants of which (d^{20}_4 =0.6980, n^{20}_D = 1.3978) corresponded to those of tetramethyl-ethylene. Besides, this hydrocarbon was identified through its dibromo-derivative and its nitroso-chloride.

Dibromo-tetramethyl-ethylene

Eight grams of fraction, b. p. 70—76°, were dissolved in 40 cm³ of chloroform and under cooling with a mixture of ice and salt a solution of 16 g of bromine in 60 cm³ of chloroform was added dropwise. After the elimination of chloroform a crystalline product was obtained having a strong camphor-like odour and m. p. 168—169° (according to literary data the dibromide of tetramethyl-ethylene has m. p. 169—170°) (⁷).

Analysis (according to Stepanov). Found: Br 67.36%. Calcd: Br 65.86%.

The nitroso-chloride of tetramethyl-ethylene was obtained under conditions described by Thiede (⁸). Like the dibromo-derivative it had a camphor-like odour and was highly volatile. Its melting point was 117—121°. According to Thiede, the nitroso-chloride of tetramethyl-ethylene has m. p. 121°.

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CHEMICAL TECHNOLOGY

HYDROCHLORIC ACID CONVERSION OF APATITES TO FERTILIZERS, RARE EARTHS AND FLUORIDE SALTS

By S. I. WOLFKOVICH, Corresponding Member of the Academy, and A. A. LOGINOVA

The apatite concentrate of Chibiny, unlike organogenic phosphorites, contains more than 1 per cent of rare earths of the cerium group, and 3.2 per cent of fluorine, being thus higher in these substances than any other of the main types of phosphorites obtained in this country.

When apatite is worked over into concentrated fertilizers and salts by means of sulphuric acid, only 20—40 per cent of its rare earths can be recovered [1]. But hydrochloric acid, like nitric acid, permits as much as 98—99 per cent of rare earths to be passed into solution.

The complex decomposition of apatite by means of hydrochloric acid according to the scheme developed by the authors, which involves a step-by-step precipitation of fluorine, rare earths and phosphoric acid, has much in common with the scheme of nitric acid conversion of apatite described in detail earlier [2, 3], and also with the hydrochloric acid decomposition of phosphorites, as investigated in the Institute of Fertilizers and Insectofungicides [4, 5].

The scheme of the process studied by the authors can be represented by the following principal equations.

The apatite is decomposed by hydrochloric acid according to the equation

$$Ca_5F(PO_4)_3 + 10HCl = 5CaCl_2 + 3H_3PO_4 + HF \qquad (1)$$

The phosphoric acid and the sesquioxides, including the rare earths, pass therewith into solution

$$R_2O_3 + 6HCl = 2RCl_3 + 3H_2O \qquad (2)$$

HF obtained according to equation (1) combines with SiO_2 to form H_2SiF_6

$$6HF + SiO_2 = H_2SiF_6 + 2H_2O \qquad (3)$$

The solution obtained as a result of the decomposition of apatite, according to equations (1), (2) and (3), is first worked over by NaCl to separate Na_2SiF_6 and then neutralized in two steps with the aid of $CaCO_3$ [or $Ca(OH)_2$] to carry down the rare earth phosphates first, and the dicalcium phosphates afterwards (the precipitate). These reactions can be represented schematically by the following equations:

$$H_2SiF_6 + 2NaCl \rightarrow Na_2SiF_6 + 2HCl \qquad (4)$$

The sodium fluorosilicate passes into solution in the presence of an excess of sodium chloride. Upon the separation of the fluorine the solution is partly neutralized with limestone up to 65—70 per cent of the first hydrogen ion of H_3PO_4.

The rare earths are precipitated in the form of phosphates

$$2RCl_3 + 5H_3PO_4 + 4CaCO_3 \rightarrow \downarrow 2RPO_4 + 3CaCl_2 + H_3PO_4 + \\ + Ca(H_2PO_4)_2 + 4CO_2 + 4H_2O \quad (5)$$

The further neutralization of the solution (5) obtained after separating the rare earth phosphates, proceeds with the formation of dicalcium phosphate according to the equation (6)

$$H_3PO_4 + Ca(H_2PO_4)_2 + 2CaCO_3 \rightarrow 3CaHPO_4 + 2CO_2 + 2H_2O \quad (6)$$

A physical-chemical investigation of the system

$$CaO - P_2O_5 - HCl - H_2O$$

was carried out by Taperova and Shulgina (6).

The isotherm of $CaO—P_2O_5—HCl—H_2O$ at 25° C consists of three saturation fields: anhydrous dicalcium phosphate, monocalcium phosphate and a double-salt—$Ca(H_2PO_4)_2 \cdot CaCl_2 \cdot 2H_2O$.

Chlorides have a salting-out effect on the phosphates of calcium. As the concentration of HCl grows from 0 to 20 per cent, the concentration of P_2O_5 in the solution falls off reaching 2.7 per cent.

Our experiments and a comparison of their results with the data of Taperova and Shulgina have shown that the solutions obtained upon the decomposition of the apatite by hydrochloric acid are not saturated with respect to CaO.

Neutralizing with calcium oxide the solution obtained by decomposing the apatite with a 15 and 20 per cent hydrochloric acid yields not saturated solutions with respect to monocalcium phosphates. This case takes place when the first hydrogen ion of the phosphoric acid is neutralized with limestone down to 50 per cent with the purpose to separate the rare earth phosphates.

Lest the monocalcium phosphate should be carried down along with the rare earths, one has to carry on the process of neutralization at above 25° C, because increasing the temperature up to 50° C shifts the region of monocalcium phosphate precipitation towards higher concentrations of calcium chloride.

The possibility of precipitating the double salt $Ca(H_2PO_4)_2 \cdot CaCl_2 \cdot 2H_2O$ by increasing the concentration of $CaCl_2$ in the solution has also been pointed out by Fox and Clark (7) in their paper on the decomposition of Tenessy phosphates with hydrochloric acid.

Taking into consideration the dense packing of the ions $PO_4^{\cdot\cdot\cdot}$ and $Ca^{\cdot\cdot}$ in the apatite and the high energy of its crystalline lattice, which accounts for its slower decomposition by acids as compared to phosphorite, we thought it necessary in the first place to study the influence of the concentration of the initial HCl on the rate of apatite decomposition within 4 to 20 per cent, and also the optimal time of interaction of the reagents at the temperatures of the decomposition process (30—50° C).

The precipitation of fluorine in the form of Na_2SiF_6 upon the introduction of NaCl in the solution has been sufficiently investigated in this Institute and elsewhere (9, 10).

The hydrochloric acid solutions were precipitated with limestone, because limestone had been found to act better than lime white. A precipitate carried down by limestone shows a better crystalline structure. It settles well, and is easily filtered off and washed, and the mass left on the filter is much dryer than that from the precipitate obtained by means of lime white.

The apatite, the limestone, the intermediate solutions and the ultimate products were analysed for the following components: P_2O_5, CaO, F, $T(R_2O_3)$, Cl, HCl, Fe_2O_3, insoluble residue and moisture, according to the methods adopted in this Institute (8).

On the basis of verified laboratory experiments the following scheme of the technological process was worked out by the authors.

The apatite is decomposed by 15 per cent hydrochloric acid. The pulp obtained upon this decomposition is decanted. From the decanted liquor, containing 0.03 per cent of insoluble residue, fluorine is precipitated by means of a NaCl solution.

After the precipitation of the fluorine the pulp is decanted; the condensed portion is subjected to filtration, whilst from the decanted portion the rare earths are precipitated. The precipitate is washed with water and dried. After the separation of fluorine the washings from Na_2SiF_6 and from the rare earth precipitate are added to the main solution, from which rare earths have been precipitated.

The precipitation of rare earths is effected by means of a finely ground (150—200 mesh) limestone. The pulp with the precipitate of rare earth phosphates is decanted and filtered. The deposit is washed with water and then worked over to rare earth fluorides.

After decantation the solution is acted upon with a limestone suspension (phase I) as a precipitant, arising from precipitation of P_2O_5 in the filtrate obtained after the first phase of precipitation. The pulp containing the precipitate is filtered; the filtrate which contains 0.2 per cent of P_5O_2 is precipitated with dry lismetone in quantities necessary for precipitation of the entire phosphoric acid (phase II).

The pulp thus obtained contains mainly limestone and a small amount of the precipitate. It is then decanted; the pulp condensed to L : S=2 : 1 is treated for the precipitation of the first phase. The decanted liquor consisting mainly of calcium chloride is a waste product which may, however, be utilized for other purposes. The precipitate after filtration is washed off with calcium chloride and dried.

The optimal conditions for the process of decomposition of apatite by hydrochloric acid are as follows: the initial concentration of the hydrochloric acid is 15 per cent; temperature, 30° C; time of the reaction, 2—3 hours. If a 10 per cent hydrochloric acid is used, the reaction time increases to 4 hours. Increasing the temperature up to 50° C considerably increases the rate of apatite decomposition by hydrochloric acid if the acid is used in low concentrations—4 to 10 per cent.

If the concentration ranges from 15 to 20 per cent, the influence of temperature is far weaker.

To attain a practically complete decomposition of apatite (98 per cent) the proportion of hydrochloric acid should correspond to the stoicheiometric norm (2 mol. HCl per one mol. CaO). The lower the initial concentration of the acid, the greater excess of it is necessary.

The proportion of fluorine decomposition in the form of Na_2SiF_6 reached 75—80 per cent when the rate of decantation of the deposit was 0.5—0.3 m/hour.

The study of the process of rare earth decomposition has shown that the quantity of the precipitate is not affected by the initial concentration of the hydrochloric acid extract, if it varies between 20 and 15 per cent of $CaCl_2$.

The rare earth phosphates begin to be carried down with a 10 per cent neutralization of the first nitrogen ion of H_3PO_4. When the degree of neutralization of the first hydrogenation of H_3PO_4 increases to 25—45 per cent, the proportion of the rare earth oxides passed into solution is 70—75 per cent.

If the degree of neutralization amounts to 80 per cent, the yield of rare earths grows by as much as 5—7 per cent. In this case the content of rare earth oxides in the precipitate is greatly reduced—from 44 down to 33 per cent.

The optimal degree of neutralization of the first hydrogen ion of H_3PO_4 is 50—60 per cent. The yield of rare earths is then 80 per cent; the content of rare earth oxides in the precipitate, 38—40 per cent.

The precipitation of phosphate extracts by limestone proceeds under conditions which are very close to those established in earlier experiments ([1], [5]). The norm of the limestone is 105 per cent with respect to the stoicheiometric norm calculated for $CaHPO_4$. The time of the interaction between the reagents is 3 to 4 hours. If the second phase of precipitation is used, 100 per cent of phosphoric acid can be carried down.

The yield of different products in a ton of the initial floatation concentrate of apatite containing 39.3 per cent P_2O_5 is as follows: sodium fluorosilicate (a 95 per cent solution), 42.7 kg; rare earth phosphates with 40 per cent TR_2O_3, 19 kg; the precipitate with 45 per cent P_2O_5 and 6 per cent moisture, 842 kg.

The expenditure coefficients per 1 ton of floatation apatite were: commercial 15 per cent hydrochloric acid, 4520 kg; common salt, 223 kg; limestone, 607.8 kg.

It will be obvious that the complex process of hydrochloric acid treatment of apatite discovered here opens good prospects for a full use of the phosphoric acid, rare earths and, to considerable extent, of the fluorine contained in the floatation concentrate of Chibiny.

This process may find commercial application in the manufacture of precipitates useful as fertilizers and, possibly, as food. In the first place its use may be recommended in those localities where hydrochloric acid is cheap or is produced in excess, *e. g.* where sodium sulphide or potassium sulphide are manufactured from chlorides, or chlorine is produced electrochemically, also in the case of chloride hydrolysis, *etc.*

In conclusion the authors desire to express their gratitude to M. S. Strepikheeva for carrying out the analyses involved in this work.

Samoilov Institute of Fertilizers and Insectofungicides.

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GEOLOGY

A QUANTITATIVE METHOD FOR STUDYING OSCILLATORY MOVEMENTS OF EARTH'S CRUST

By A. B. RONOV

(Communicated by V. G. Chlopin, Member of the Academy, 10. XI. 1943)

The proposed quantitative method for studying certain geological phenomena of the past is based upon the modern views concerning the processes of formation of thicknesses of deposits ([1], [2]). Along with this, the method is backed with the latest achievement in radium geology which has worked out the absolute time scale ([4], [5], [3]). The further development of this method is connected with the precision of the scale of absolute time giving in so far only the duration of entire periods.

The application of the method to the Urals, Big Caucasus, Donetz basin and Volga-Ural region has shown that on the whole the results obtained by its means give correct values characterizing the processes of the past of the Earth.

1. A Volumetric Method for Studying the Descending Movements of the Earth's Crust

The direct relationship existing between the thickness of the deposits and the subsidence of the Earth's crust has been convincingly proved by V. V. Belousov ([1], [2]).

The thicknesses of the deposits are only one of the three coordinates characterizing the subsidence of the Earth's crust. The sinking should naturally find its fullest expression in the volumes of the rocks accumulated. In the first and sufficiently close approximation the volume of the subsidence of a certain area of the Earth's crust is equal to that of the sedimentary and effusive rocks accumulated here (if the latter are present in the section). The chief object is thus to find a method for measuring the volumes of the deposits.

The total volume of the sedimentary and volcanic rocks is determined with the aid of isopachymetrical schemes representing the plan of distribution of thicknesses of the deposits.

In passing from the total volumes to those of the sedimentary and volcanic rocks separately one is helped by the geological maps permitting to establish the quantitative (areal) proportion between the effusive and the sedimentary rocks on the Earth's surface.

Under conditions of complex folding the relations between the areas of the surface exposures of conformably occurring strata established over a large area, are approximately equal to the ratio of their volumes.

If a sufficiently detailed map of the territory investigated be taken, and the areas of the development of various rocks of the same geological age be measured upon it, upon a subsequent calculation of the ratio of these areas to each other the proportionality coefficient will be obtained, determining not only the relation between the exposures, but also that between the volumes.

Having established the proportionality coefficients for those epochs for which the total volumes of deposits were previously calculated from isopachymetrical schemes, it will be possible to determine the particular volumes of the sedimentary and effusive rocks.

In order to obtain a statistical mean characterizing the region as a whole, the total volume of deposits should be referred to unit area (1 km²). Recalculation is made for a successive series of stratigraphic units.

The values obtained characterize the average course of accumulation of deposits for a given portion of the Earth's crust. From these values the curves showing the accumulation of deposits in time are plotted to illustrate the above-said. A graph is given, representing the amount of subsidence of the Earth's crust and the accumulation of carbonate and terrigenous deposits of the Big Caucasus during the Mesocenozoic (Fig. 1).

An idea of the average rate of subsidence of the Earth's crust is gained from the total volumes of the sedimentary rocks and effusives deposited per 1 km² of the territory investigated during a million years of each epoch. The absolute duration of the epoch is established from the existing scale of absolute geological time recording ([3–5]).

The same procedure is applied to the measurement of the absolute intensity of volcanism, that is the average volume of effusives accumulated during a million years at a given epoch per 1 km². The curves of Fig. 2 show the rate of subsi-

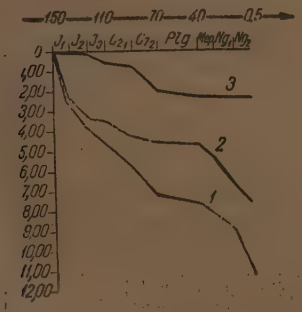

Fig. 1. Amount of subsidence of Earth's crust and accumulation of deposits in geosyncline of Big Caucasus: *1*, amount of subsidence; *2*, terrigenous deposits; *3*, carbonate rocks

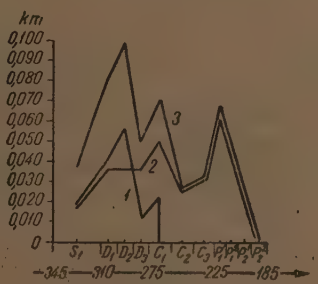

Fig. 2. Rate of subsidence of the Earth's crust and rate of accumulation of sedimentary rocks is Uralian geosyncline: *1*, intensity of volcanism; *2*, rate of accumulation of sedimentary rocks; *3*, rate of subsidence of Earth's crust

dence of the Earth's crust, the rate of accumulation of sediments and the intensity of volcanism during the Hercynian cycle of development of the Uralian geosyncline.

An analysis of these curves, as well as an analysis of analogous curves plotted for the Big Caucasus has made it possible to establish the existence of a direct functional relation between the intensity of volcanism and the rate of subsidence of the Earth's crust. Namely, the epochs of an intensive manifestation of effusive volcanic activity are synchronous with the epochs of the maximum velocity of subsidence of the Earth's crust, and reversely, the epochs of volcanic quiescence coincide with those of a slow subsidence of the Earth's crust.

2. A Volumetric Method for Studying the Ascending Movements of the Earth's Crust

The studying of upheavals is a more complicated matter, since the areas of elevation are at the same time areas of erosion and, consequently, no accumula-

tion of deposits takes place within them. However, there is a different method for studying elevations. We mean the study of the relation between elevations and subsidences in time. It is known that in the Earth's crust there always exist side by side zones of subsidence and zones of absolute and relative elevation. In the latter the process of accumulation of substantial thicknesses of deposits proceeds at a slower rate than in the zones of subsidence (1). In studying the geosynclinal area as a whole. *i. e.* comprising both the areas of upheaval and those of subsidence, it is possible to speak of a predominance of elevation or, inversely, of subsidence of the Earth's crust, and to estimate quantitatively the relation between these movements. If the volume of all the rocks corresponds to the volume of subsidence of the Earth's crust, the volumes of the terrigenous deposits supplied from the area of upheaval to the area of subsidence correspond to the volume of elevation of the Earth's crust. Hence, when studying in time the relation between the volumes of terrigenous deposits and the total volumes of deposits, we are studying the ratio of the elevation of the Earth's crust to its subsidence. Let us term this ratio the elevation coefficient.

The value of the coefficient may vary from 0 to 1 and even somewhat exceed unity. The latter occurs at the end of the cycle, when the supply and accumulation of terrigenous material proceeds at a more rapid rate than does the sinking of the Earth's crust. As a result of this, the surface of the sediment rises and attains a level above the surface of the sea (continental deposits). In the range of variations of the elevation coefficient the following characteristic intercepts and points may be distinguished: when the elevation coefficient is equal to zero, the Earth's crust is subject only to descending movement. Within a range from 0 to 1 the subsidence of the Earth's crust predominates over the part played by the ascending movements, becoming greater as the elevation coefficient increases. With an elevation coefficient equal to 1 or above 1, there sets in a prevalence of the elevation of the Earth's crust, which terminates at the end of the cycle in a general upheaval of the land.

The curve in Fig. 3 shows the interaction between the ascending and the descending movements during a part of the Caledonian and throughout the Hercy-

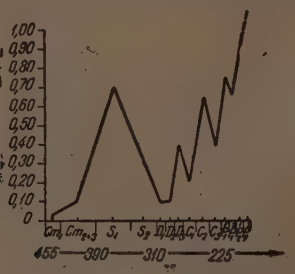

Fig. 3. Ratio of upheaval and subsidences in Uralian geosyncline

nian cycle of development of the Uralian geosyncline. Within the Hercynian cycle the author succeeded in establishing small cycles represented by curves of the second order. Both the large and the small cycles are characterized by approximately the same sequence of its evolution. At the beginning of the cycle a considerable part is played by ascending movements; in the middle subsidence of the Earth's crust predominates, and the end of the cycle is marked by a new increase in the forces of elevation. From an analysis of the curve in Fig. 3 it will be evident that the Caledonian cycle is not completed and, as it were, passes into the second; Hercynian, more complete cycle of development of the Uralian geosyncline. The Lower Silurian peak of the curve shows that there was probably no general upheaval of the land at the end of the Caledonian cycle, and the increase of the ascen-

ding movements was manifest only in local uplifts, the erosion of which supplied material to the Lower Silurian clastic complex of rocks.

Along with the development of an interaction of oscillatory movements of different signs between the geosynclines of the Urals and of the Big Caucasus, an evolution of facial conditions proceeded. The quantitative data obtained indicate that at the beginning of the Hercynian and the Alpine cycle, in the two geosynlines there predominated terrigenous and effusive deposits, respectively; in the middle, carbonate rocks became of primary importance; and at the end of the cycles, terrigenous and continental facies.

Radium Institute.
Academy of Sciences of the USSR.

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PLANT PHYSIOLOGY

ON THE SYNTHESIS OF RUBBER IN PLANTS

II. ON THE PROBABILITY OF RUBBER MIGRATION IN PLANTS

By A. A. PROKOFIEV

(Communicated by A. N. Bach, Member of the Academy, 5. VI. 1943)

Many hypotheses have been advanced in order to locate the formation of rubber in plants; in all of them much attention was given to the movement of the rubber within the plant. According to some of the writers ([1], [2]), the rubber is formed in the assimilating tissues of the leaf; hence it is conveyed along the laticiferous vessels, or along some other tissues, to the axial organs and to the roots, in particular. While discussing in a previous paper ([3]) the possibility of the rubber being transported along the sieve-tubes and parenchymatous cells, the author answered this question in the negative. But the matter appeared much more involved still in the case of the migration of rubber along the laticiferous system. Numerous experiments involving the girdling of the plants possessing laticiferous vessels ([4]), the migration of dyestuffs and salts ([5],[6]), as well as direct observations made on intact laticiferous vessels ([7]) give rather conflicting results. In the author's opinion this is due to the fact that the methods applied disturb strongly the normal state of the plants, thus leading to the most variable distortions of the actual picture. For this reason we adopted an essentially different method which did not interfere with the plant's life activity.

The latex of various plants is known to contain rubber in the shape of well-defined structures, *viz.* globules. These globules vary both in shape and size, according to the age of the latex tube. In an adult latex tube, and, consequently, in an adult plant, the shape and size of the rubber globules is a well-expressed and specific character. Thus, for example, the rubber globules of kok-saghyz are spherical in shape, ranging in size between ultramicroscopic values and 1—2 μ across. In tau-saghyz they are club-like and reach 10—12 μ in length, while in krym-saghyz they are the shape of elongated rods, reaching 12—15 μ in length*. These peculiar characters of the globules which distinguish the various rubber-plants from one another, are stably preserved whatever the mode of cultivation. Neither are they affected by differences in soil or climate. The shape and size of a rubber globule is thus an excellent character, according to which one may safely determine the origin of the latex. It is this circumstance that we decided to turn to profit when thinking over the question, whether the latex migrates along the laticiferous vessels.

The various latex-producing plants were so combined in the grafts that the stocks and scions would sharply differ according to the kind of their latex globules. The procedure of grafting was the following. Roots of 2—3-month-old plants were dug, freed from earth and cut into bits 3 cm large. After this two bits equal in diameter were taken from the plants to be united (say, kok-saghyz and krym-saghyz). The scion and stock roots were cut with a sloping cut and joined, after which the joint was bound in flax or raphia fibres. The cuttings thus obtained were planted into pots filled with sand. The polarity of the roots should

* The rubber-globules of the latices of tropical rubber trees (hevea, manihot, ficus) also differ as to shape and size.

be strictly observed, lest on the lower end of the stock leaves would appear, instead of the roots expected. It is also very important that the two surfaces of the cuts coincide with each other. If one fails to observe this condition, leaves may appear on the upper part of the stock.

In some 6—7 days upon grafting both stock and scion form a callus, and after 10—12 days they form roots and leaflets, respectively. In the case of grafting within the genus *Taraxacum*, there is a perfect union between stock and scion, so that the demarkation line can hardly be distinguished. Sometimes the boundary between stock and scion can be recognized by the colour of their covering tissues only; so the cork in krym-saghyz is darker than in kok-saghyz and in the dandelion. According as the grafted plant continues growing, no difference can be observed between the two partners, save in the parts taken for the graft. The new growing tissues develop as if they belonged to a single plant. The further growth and development of the grafted plants proceeds in an absolutely normal way. In 1941, when we were able to obtain our seedlings from a greenhouse, the grafted plants bore fruit within the same year. The seed obtained from the grafts proved of good quality and, when sown, sprouted at one.

We were greatly interested to learn whether the laticiferous vessels of stock and scion «healed-in», *i. e.* whether a continuous laticiferous system was formed that would connect the aboveground parts of the grafted plant and its root. Such a system would warrant a transfusion of the latex from the leaves on to the root, provided such a transportation of latex actually exists, as assumed by some writers. Microscopic examinations of grafts have shown that such a contact between the latex system of the stock and that of the scion takes place, indeed. But nevertheless, we failed to state the very fact of penetration of the scion's latex into the stock, and *vice versa*. Thus, for example, the examination of the latices taken from stocks and scions in the grafts of kok-saghyz to krym-saghyz, has shown that the latex of each of the components preserved the typical rubber globules*. In the graft referred to above, kok-saghyz has invariably been found to contain small spherical globules peculiar to it, whereas krym-saghyz showed comparatively big, rod-like globules. And the picture was always the same, irrespective of whether a given plant had been used in the graft as a stock or a scion. The same regularity has been discovered for plants obtained by grafting krym-saghyz to non-rubber-bearing dandelions (*Taraxacum bessaraticum*, *T. multiscaposum*, *T. brevicorniculatum*). All those dandelions have globules of a spherical shape, which are preserved both in the area of the graft and in the laticiferous vessels formed anew. And we failed to record a single case of penetration of the typical krym-saghyz globules into the latex tubes of the dandelions.

Along with the grafts performed within the genus *Taraxacum*, we also examined grafts between tau-saghyz and various other *Scorzonerae*, as well as grafts between tau-saghyz and *Tragopogon*. These combinations have kindly been put at our disposal by comrade Semashko, plant-breeder at the Kaunch Experiment Station of the Rubber-Plant Institute. The method of grafting applied by Semashko (cleft-grafting) differed materially from that described above. Plants of various age were taken as stocks. Not infrequently stocks or scions were taken from plants 2—4-year-old. It has been shown by anatomical analysis that cleft-grafting, though resulting in most of the cases in a union between stock and scion, did not altogether preclude, however, the possibility of an independent existence of the parts grafted. Indeed, in most of the cases the stock had roots of its own, while the scion formed a leaf-rosette. Besides, the use as scions of rather large root-sections brought about a depression of the stock. Thus, for example, the root part of the scion in a number of combinations was found to be much thickened, while the stock had a rather poor aspect. This inhibited condition of the stock was espe-

* The method of microscopic examination of the latex was as follows: a small quantity of latex excreted by a tapped organ was mixed with water; a drop of the strongly diluted latex obtained was spread over an object-glass in a very thin layer. When dried, the preparation was examined at a 100-fold magnification, the rubber globules being drawn with the aid of Abbe-Zeiss camera lucida.

Shape and Size of Latex Globules in Various Species of *Scorzonera* Genus

Species	Age	Shape of globules	Maximum size of globules in μ (diameter in spherical, length in elongated globules)
Scorzonera tau-saghyz	4 months	Dolioform and club-like	5.0
» » »	1 year	Club-like	15.0
» » »	3—4 years	»	15.0
» *hispanica*	4 months	Spherical	2.0
» »	2 years	»	3.0
» *saesillus*	2 »	»	1.0
» *poposa*	2 »	»	1.5
» *tragopogonoides*	2 »	»	2.0

cially conspicuous in the case where the rapidly growing *Scorzonera hispanica* served as a scion, while the part of stock was played by the slow-growing tau-saghyz. In this case the scion-root of *S. hispanica* was found to be thickened considerably, while the root of tau-saghyz did scarcely thicken at all. The main part of the assimilates supplied to the leaves of the scion were apparently captured on their way by the root of *S. hispanica*. We consider therefore the method of cleft-grafting which gives no harmonious development of the parts grafted, inadequate for solving the problem here discussed. Yet, in so far as this method also gives in many cases a good union of the components, it appeared of some interest to extend our investigations to grafts performed within the genus *Scorzonera*. A preliminary examination of the latex of the poor rubber-yielding *Scorzonerae* and of tau-saghyz gave the following picture (see table).

As may be seen from this table, the two components of the graft, tau-saghyz and *Scorzonera* which is a poor rubber-producer, form a latex which differs strongly as to the shape and size of the rubber globules.

We examined both annual and perennial grafts between tau-saghyz ant the *Scorzonerae* listed in the table above. Irrespective of their position in the graft, the *Scorzonerae* invariably contained in their latex exclusively spherical rubber globules. The latex of tau-saghyz, on the other hand, contained exclusively club-like globules which are peculiar to this plant. In a single graft only (scion from tau-saghyz, stock from *Scorzonera hispanica*) the latex of the tau-saghyz scion near the place of union showed spherical globules instead of the typical club-like ones. The microscopic examination of this plant revealed, however, that all the mature laticiferous tubes of the scion had died away, and that the latex was exuded by young, newly formed laticiferous vessels only, which in tau-saghyz always contain rubber in the shape of small spherical globules.

From an analysis of the results obtained we arrive at a conclusion that the laticiferous tubes do not play in the plant the part of conveying channels, for the latex they contain remains immobile under normal conditions. Hence it follows that the synthesis of rubber takes place in the same tissues in which this substance is accumulated. In the root-rubber plants the accumulation of rubber is dependent on the activity of the laticiferous system of the root, and is by no means due to the afflux of the latex from the leaves.

Institute of Plant Physiology,
Academy of Sciences of the USSR.

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ZOOLOGY

CONTRIBUTION TO THE TAXONOMY OF *MICROTUS RATTICEPS* KEYSERL. ET BLAS.

By S. I. OGNEV

(Communicated by L. A. Orbeli, Member of the Academy, 22. XI. 1943)

This note should indispensably be prefaced with some remarks on the nomenclature of the species described. In the recent literature on the subject this vole is given the name of *Microtus oeconomus* Pallas, who had been the first to describe it (1). His description is rather concise, however, and lacking precision; the distribution of the species is indicated to extend «throughout Siberia up to Kamchatka». In his subsequent book (2) Pallas gave a much more detailed description of this vole. Its geographical distribution was indicated by him by the following words: «primum omnium in campis Ischimensibus et ad Irtin ad tentionem meam exciturunt». On the ground of this information it is West Siberia that should be regarded as the terra typica of *M. oeconomus* Pall., and more precisely, the valley of the Ishim river. A scrupulous comparison of the description of «*Mus oeconomus*» as given in «Novae Species» with the *Microtus oeconomus* portrayed by the more recent authors makes one inevitably conclude that Pallas understood by his description some other vole whose characters do not coincide with those of *M. oeconomus* as it is understood in the recent literature.

In fact, according to Pallas' description, «*Mus oeconomus*» is much like *Microtus arvalis*, except for a more yellowish fur which is somewhat darkened along the middle of the back by an admixture of black hairs, a smaller head and a very short tail, making up about a quarter of the total body length. The complete divergency between the original description of *Mus oeconomus* and the conceptions of the later writers was stressed by A. Jakobi (3). As far back as in 1881, in his well-known «Monographs of voles» I. S. Poljakov stated without any ambiguity that Pallas and Eversmann had incorrectly described the rodent *Arvicola oeconomus* as having a tail as short as $^1/_4$ of the body. According to I. S. Poljakov, Eversmann doubtless confounded *Arvicola oeconomus* (as understood by I. S. Poljakov and the subsequent writers) with *A. eversmanni* (=*Stenocranius gregalis eversmanni* Poljak.). Eversmann's collection comprised several specimens of the latter vole, determined by him as *Arvicola oeconomus*. It seems most probable that Pallas himself has also founded his diagnosis of «*Mus oeconomus*» upon characters belonging to the west-siberian specimens of *Stenocranius gregalis*. But he probably had at his disposal «genuine *Mus oeconomus*» (=*Microtus ratticeps*) as well; the occurrence of «var. *kamtschatica muris oeconomi*» makes this supposition still more plausible (2).

Because of this obscurity and confusion it would be right to take as the specific the name of the *Microtus ratticeps* Keys. et Blas. (1841). Terra typica—region of Velikiy Ustjug (on the N. Dvina, in the former Vologda gov.). As to the Kamchatka subspecies, the most appropriate name for it is *Microtus ratticeps kamtschaticus* Poljakov ex Pallas.

We give here a list of the forms of the *Microtus ratticeps* which occur at the present time within the boundaries of this country; some of them are described here for the first time.

1. *Microtus ratticeps ratticeps* Keys. et Blas. (1841).

Distributed over the northern regions of the European part of this Union, from Lapland, Arkhangel district to Pskov, Novgorod, Vologda, Kalinin (Tver), Kirov (Viatka) districts.

2. *Microtus ratticeps stimmingi* Nehring (1899).

Terra typica—East Prussia, vicinity of Brandenburg. Distributed (aside from the type locality of the subspecies) in the Near-Baltic region, in White Russia (Minsk, Moghilev districts), Smolensk, Kaluga, Moscow, Riazan districts, Middle Volga region.

3. *Microtus ratticeps petshorae* subsp. nov.

Type locality: N. S. 25980 (No. 229), ♀, 16.IX.1929, town Nizhnyaya Pesha (the Cheshskaja bay). A. I. Chirkova (Coll. of the Zool. Mus. of the Moscow University, flat skin without skull). Aside from the type, 39 specimens were examined, obtained from the following regions: Nizhnyaya Pesha, Cheshskaja bay, gulf Indiga, Perm district. To the same subspecies seems to belong a specimen (alcohol preparation) from the region of the mouth of the river Chizh, south extremity of Kaninskaya Zemlya.

D i a g n o s i s. Colour of summer fur much lighter not only than the in nominal race, but also in *M. r. stimmingi*; the reddish tint is also more pronounced. On the back there is appreciable admixture of brown-red and red hairs (of a colour intermediate between sayal-brown and micado-brown, pl. XXIX, Ridgway, 1912). The hairs on the sides show a pronounced yellowish tint. Total body length of adult individuals ranges from 123 to 142 mm, length of tail (without terminal hairs) 41—47 (this is the maximum tail length recorded in one specimen only, as a rule it attains 50 mm); length of hind foot (without nails) 18.5—20 (usually about 19). Total length of skull 27.5—28; width of zygomatic arches 15—16; length of upper set of molars 6.3—6.4 mm long.

T a x o n o m i c N o t e s. When examined in series, voles of this subspecies can easily be distinguished from the darker nominal race by the reddish tinge of their fur. As already noticed, they are lighter in colour than the Middle-Russian *M. r. stimmingi* and probably larger than the latter.

The area of distribution of this species has as yet not been precisely determined. *M. r. petshorae* may possibly be encountered in the Timan tundra and in a part of the Bolshezemlya tundra; at the Kaninskaya Zemlya and south-east it is likely to extend to the northern parts of the Middle Urals (Perm district).

4. *Microtus ratticeps uralensis* Poljakov (1881).

Type locality: Orenburg district. The vole is distributed in Bashkiria, in South Trans-Urals, Western Siberia, in the Semipalatinsk province and Minusinsk region, at the Salair mountain-range.

The description given to this race by I. S. Poljakov from a single specimen caught at Orenburg lacks precision. This subspecies can briefly be portrayed in the following way: a) Colour of back appreciably darker than in Middle-Russian *M. r. stimmingi*, though lighter than in the nominal race. Many of the uralian specimens display a darkening of the fur at the middle-back area, at the flanks it is lighter and with an yellowish-rusty tinge. b) Skull relatively small (total length does usually not surpass 27 mm; zygomatic arches about 15.2 mm wide). c) Tooth sets not very long (length of upper molars usually about 6.1—6.2 mm). 56 specimens examined.

5. *Microtus ratticeps altaicus* subsp. nov.

Type locality: S. 16737, ♀ sen., Lake Djulu-Kul, Chulyshman, Altay. A. M. Kolosov (Collection of the Zoological Museum, Moscow University). 137 flat skins and skulls examined.

D i a g n o s i s. The colour of the back fur somewhat dull, greyish, as compared with the nominal race; at the sides of the body there is a slight reddish tinge. Skull rather large, but tooth sets moderately large as compared with the

total dimensions of the skull. The structure of m^3 is characterized by an elongated last enamel loop and by its arched outlines.

Body length comprising head, 108—142 mm (usually about 125—130 in mature and old specimens); tail length 42—55.5 (usually about 46); length of hind foot 15.9—20 (usually about 17.6); total length of skull 26.8—29.9 (usually about 27.6); width of zygomatic arches 14.9—17 (usually about 16); length of upper tooth set 5.8—6.7 (usually about 6.3).

This form enjoys wide distribution in the Altay mountains.

6. *Microtus ratticeps montium—caelestinum* subsp. nov.

Type locality: S. 10951, ♀ ad., 19.III. 1936. Djungar Alatau, Terecty valley. Aside from the type, 21 specimens examined.

Diagnosis. Fur of the back dark, black-brown (fuscous and fuscous-black, Ridgway, pl. XLVI), somewhat mottled with brown-rusty admixture, due to the presence of rusty tinge in the subterminal parts of some of the back hairs. The flanks covered with black-brown hairs, often showing a rusty tinge. The hairs of the belly vary from a dull brown-grey to a dirty greyish-white. The dorsal side of tail black-brown, the abdominal, grey-brown.

Skull relatively small, zygomatic arches narrow; molar set shortened. m^3 has an elongated enamel loop bent inward (a character in common with the Altay specimens, but still more sharply pronounced). Body length 110—133 mm; tail length (without terminal hairs) 38—45; hind foot (without nails) 17—19; total length of skull 25.4—27.3; condylo-basal length of skull 25.7—28.2; zygomatic arches 14—15.2; skull capsule 14—14.8 long; length of upper set of molars 5.8—6.2 mm.

One of the best differentiated subspecies which stands out clearly when examined in series.

7. *Microtus ratticeps oeconomus* Poljakov (1881) (nec apud Pallas!).

Type locality: north-western coast of lake Baikal (I. S. Poljakov caught these voles at various localities, the type specimen should therefore be chosen arbitrarily. We fixed our choice upon the specimen from Baikal as the one first listed in the description and body measurements).

Diagnosis. Back fur dark, black-grey-brown, with a very slight pale-yellow tinge (buffy-brown, Ridgway, pl. XL). Hairs of the belly grey-white, with a pale-yellow tint of various intensity. Some of the specimens have abdominal hairs of a mouse-grey, with a reddish tinge (hair-brown, pl. XLVI, with a tinge of drab or buffy-brown, pl. XL).

Skull large, zygomatic aches wide; set of molars rather short. In a number of specimens m^3 has four salient angles at its inner side, the two posterior angles are appreciably elongated and are separated by a deep re-entrant angle. The outlines of the posterior salient angles are usually rounded, especially in the last of them.

Body length (in fully mature and in old individuals) 126—140 mm; tail 40—55 (usually about 46—47); hind foot 16—21 (usually about 19); total length of skull 27—30.5 (usually about 28.5); condylo-basal length 26.9—30.9 (usually about 28.7); width of zygomatic arches 15—17 (frequently 15.5—15.6); length of skull capsule 14.5—16.5 (usually 15.6); length of upper set of molars 5.9—6.7 (usually 6.4 mm).

A well differentiated subspecies with a dark-coloured, thick and soft fur, with a large skull and rather short molars, m^3 showing certain particular traits. A total of 52 flat skins with skulls and 23 alcohol preparations examined.

8. *Microtus ratticeps hahlovi* Scalon (1935).

Type locality: settlement Zerkovenskoe on r. Taz, 65° N. l.

This subspecies is distributed along the lower reach of the Taz river, in the region of Nizhnyaya and Podkamennaya Tunguska, at the Taimyr peninsula, in the region of the lower reach of the river Khatanga, near the mouth of Lena river (up to the 71° N. l.). It seems probable that the specimens from Yamal (Schuchya river) also belong to this subspecies.

Synonymous with this form are: *Microtus oeconomus kjusjurensis* Koljushev (1935), type from settl. Kossiur, right bank of Lena river; *M. oec. naumovi* Stro-

ganov (1936), type from the river Khatanga; *M. oec. anikini* Egorin (1939), from the village (yurt) Sidelnikovo, right bank of the river Vasjugan; 16 specimens have been examined by the present author.

9. *Microtus ratticeps koreni* Gl. Allen (1914).

Type locality: neighbourhoods of the town Nizhne-Kolymsk. 21 specimens from the valley of Kolyma river have been examined by the author.

10. *Microtus ratticeps shantaricus* Ognev (1929).

Type locality: mouth of r. Yakshina, island B. Shantar. The specimen is owned by the Zoolog. Museum of the Moscow University. Moreover, 11 specimens have been examined from the island B. Shantar, from the valley of Penzhina river, from the coast of the Ulbansky bay at the Okhotsk Sea.

11. *Microtus ratticeps kamtschaticus* Poljakov (1881).

Type locality: Kamchatka peninsula. This subspecies is widely distributed all over Kamchatka, in the Anadyr province and along the Koryazk shore.

Taxonomic Notes. In 1778 Pallas entered in his «Novae Species Quadrupedum» (2) the «varietas *kamtschatica Muris oeconomi*», and specified certain characters peculiar to its coloration. As pointed out above, according to the description of the «*Mus oeconomus*» it is probably nothing else than *Stenocranius gregalis*. Therefore the kamchatskian subspecies should for a purely formal reason bear a name with reference to I. S. Poljakov rather than to Pallas, since it is the former of the two writers who had actually described the subspecies of *Microtus ratticeps* under the name of «*Arvicola kamtschatica*».

Microtus tshuktshorum Miller (1899), described from the region of Plover bay, is synonymous with the Kamchatka subspecies. As for *Microtus operarius* Nelson from the western shore of Alaska (St. Michael), it is much similar to *M. r. kamtschaticus* Poljakov, being specifically identical with *M. ratticeps*, as seen from a direct comparison of the material available.

M. V. Lomonossoff State University.
Moscow.

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EXPERIMENTAL MORPHOLOGY

ORIGIN OF PIGMENT CELLS AND VISCERAL CARTILAGE IN TELEOSTS

By G. V. LOPASHOV

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The structure of pigment cells in fish varies enormously; no less variable are the patterns formed by them both in different species and in different genetical lines (*). It is for this reason that they offer an especially convenient material for studying the inheritance physiology. Their origin in fish has, however, not been investigated by any accurate method; the same is true of the origin of the visceral cartilage which in the rest of vertebrates springs from the same source as the pigment cells, *viz.* from the neural folds. Unlike this, in fish they seemed likely to differ as to their origin, for in most of the fishes the nervous system is laid down as a cord, and is lacking the stage of neural plate with folds. Experiments aiming at the solution of this problem were started in spring 1941. The material consisted mainly in embryos of the groundling (*Misgurnus fossilis*) and also of the loach (*Nemacheilus barbatulus*) and perch (*Perca fluviatilis*). The operations were carried out under sterile conditions. After operation the embryos were kept for some time on a substrate of 2 per cent agar. A total of 126 operations were made. 69 embryos were found fit to serve our subject (part of the material perished while transferred to active feeding). The embryos were fixed 6—10 days after operation.

1. Removal of Areas of Brain Anlage in Groundling. The operations were carried out within the stage of the closed blastopore and until the stage of the «pear» (20 somites), and even in later stages within the region of the tail. Bits were removed either from the whole cord, or, in the head region, only from its upper part (roof). As made evident by Table 1, the operation mostly resulted in a total absence of pigment cells, or in a reduction of their number. A rather high percentage of partial or complete restoration of pigmentation should be accounted for by the fact that the epidermis often shrunk together, making the ends of the neural cord also approach each other. The function of the brain as a whole was also restored on many occasions. When the pigmentation was reduced, the poorly pigmented region became gradually filled with melanophores in later stages.

In the case of the removal of the roof of head-brain, the cartilaginous skeleton was sometimes found to be also underveloped (3 cases), but it was never missing altogether. If there was no neural cord within the field of operation, there was no development of the dorsal fin, while at the tail the fin developed even in the absence of neural tube and pigmentation.

2. Grafting of Brain Pieces under the Epithelium of the Yolk-Sack. In the species under experiment, especially in groundling and loach, the yolk-sack is small and it vanishes soon; the grafts are found to be on the belly wall, within the region of the heart, or on the flank.

Table 1

Results of Removal of Brain in Groundling Embryos

Operated region / Result	From eye to 3rd somite I	Anterior body part (1—10 somites) II	Middle body part (8—20 somites) III	Caudal part and tail (beyond 20th somite) IV	Total
No pigment	3	4	6	6	19
Reduced pigmentation	—	3	3	3	9
Normal pigmentation	3	3	2	2	10
Total number of operations	6	10	11	11	38

The grafts were either homoplastic (groundling→groundling, perch→perch), or xenoplastic ones (loach→perch). In the stages examined, the groundlings show but coarse stellar black-brown melanophores with branching outgrowths of medium length and a developed central body. The loach display melanophores of somewhat smaller size; tlese are very delicate, brown in colour, almost lacking outgrowths, with an edge of irregular outline. Besides they develop a great number of small xanthophores of an intense lemon-yellow colour, angular in shape. In both species a dense network of pigment cells covers the back and the upper parts of the sides, while in the perch at the initial period of development separate black pigment cells appear on the yolk-sack only, along the lower edge of the somites and of their boundary lines. They seem to consist of non-branching outgrowths, and are very few in number. The perch is therefore no convenient material for finding out the origin of the pigment cells, unless they are used as hosts in the case of interspecific grafts. Bits for grafting were taken from the anterior (I—II) parts of the body.

Table 2 makes it evident that the number of pigment cells which invariably developed around the grafted brain was so large as to make their network still denser than in normal fish larvae.

Table 2

Development of Pigment Cells in Grafts of Brain and Mesoderm

Around the transplantate / Graft	Brain (groundling →groundling)	Brain (loach → perch)	Mesoderm (groundling → groundling)
Numerous pigment cells	9	5	1
Sparse pigment cells	2	1	2
No pigment cells	—	—	3+1*
Total number of grafts	11	6	7

* Loach → perch.

In the case of grafts from loach to perch, there was a development of pigment cells typical of the loach, which proceeded simultaneously with that of the control; the xanthophores outstood with particular distinctness. Towards the 7—9th day the grafts started rapidly to decrease in size; their pigment cells vanished altogether, which seems to be due to the setting-in of the reaction of tissue incompatibility.

As a rule, it was the roof of the brain that served for the graft, but separate grafting of bottom and roof of the brain from the head (I) region of stage of 3—5 so-

mites were also practised in groundling. Remarkably enough, pigment cells were in such cases formed not only from the roof, but from the bottom of the brain, as well, though in somewhat lesser numbers. In case the roof was grafted, the pigment cells settled mainly on the graft itself; if it was the bottom of the brain, they were found to be located somewhat apart, around the graft. It seems probable that unlike the upper part of the brain which under natural conditions is covered with pigment cells, the lower part of the brain has an inhibiting effect upon the development of pigment cells; this may be examplified by the picture observed in fowl, where the development of pigment cells is suppressed in the case of grafts made into the brain cavity ([3]).

In three of the cases, the grafting of the head (I) part of the brain also gave rise to a cartilage located near the brain, and this irrespective of whether the roof or the bottom of the brain was used for the grafting. In one of those cases the graft was made in the perch homoplastically. In another case (ZR 187) a second brain fragment has been found next to the brain, containing two auditory vesicles; this seems to be due to induction. There were no cases of formation of dorsal fins, which may possibly be connected with the fact that all our grafts were taken from the region of the 8th somite. In one of the cases (ZR 164), where the brain happened to be grafted on to the region of the anterior end of the preanal fin, the latter was found to have split throughout all its length. There is no clearness about the determination of the unpaired ventral fin; it may be assumed therefore that in contrast with the dorsal one, this fin arises within a zone of inductions opposite to those which take place in the upper part of the back. This is probably why fin formation occasionally occurs at a certain distance from a brain graft which carries in factors of the upper crest of the body.

3. Grafting of Mesoderm under the Epithelium of the Yolk-Sack was attempted in order to test the mesoderm as a possible source of pigment cells and visceral cartilage. I grafted: chorda+somites (groundling→groundling, loach→perch), or chorda+somites+lateral plates (perch→perch). It may be seen from Table 2 that in most of the cases pigment cells failed to originate upon the graft. In those three cases where they did so, the graft was overlain by a neural tube which had probably been induced by the mesoderm. In one case (ZR 140), where the graft had been taken from the region of the 1—8th somite and lay in the anterior part of the body, there were two auditory vesicles at either side of the brain. The grafts contained no cartilage. None of the ten grafts of mesoderm resulted in the form of dorsal fins.

In all the three cases of grafting of the mesoderm of 1—5 somites, we observed in perch the formation of pectoral fins; in two cases there were two; in one case, a single pectoral fin. According to their size they approximated those of the host, and responded to stimulation by intense movements. This phenomenon is analogous to the development of forelimbs in the case of the grafts of the mesoderm in amphibians ([16]). There was not a single case of formation of fins in groundling in response to grafting with the anterior axial mesoderm; it is not clear whether such a result should be put down to the absence in these grafts of lateral plates, or to species—specific features.

4. Apart stands a case where a part of invaginated axial mesoderm in the stage of the closure of the blastopore was grafted on to the yolk sack in groundling (ZR 136). The graft gave rise to the development of brain, of numerous pigment cells and of a large quantity of cartilage, not only in the shape of rods, but also as a compact mass, exceeding in volume the visceral cartilage of the control. That these structures were derivatives of the graft rather than products of induction, was evidenced by the fact that mesoderm cells were entirely lacking among them. Hence it is obvious that this case gives no ground to conclude as to a mesodermal origin of the cellular types developed. The whole way of development of the graft might well have changed under the conditions of isolation from the embryo; the visceral cartilage and the pigment cells arose as a part of the new complex: the anlage of the head part of the brain into which the tissue of the grafted mesoderm had transformed.

Discussion and Conclusions

1) Like in amphibians ([4], [5], [7], [10], [11], [15]), birds ([1], [2], [3], [14]) and mammals ([13]), the pigment cells and the visceral cartilage in bony fish arise as derivatives from the anlage of the nervous system rather than the mesoderm. The fishes investigated by me can be told apart from the other investigated vertebrates by that these types of cells are formed in them from the anlage of the nervous system as a whole. It does not seem impossible that they arise from the bottom part of the brain under the grafting conditions, which gives no way to further inducting influences of the mesoderm; this accounts for the formation of such types of cells as require a different and possibly smaller total influence to come into being. But it is more probable, however, that the data obtained will also hold in the case of normal development. They should then be brought into connexion with the fact that in the fish species investigated by us there is no separation of the anlage of the nervous system into a neural plate and the neural folds.

2) The axial mesoderm of the groundling, which already underwent invagination, when grafted on to the yolk sack, is able to transform into derivatives of the neural anlage, *viz.* into brain, pigment cells and visceral cartilage. It is known that in some fish species (*Salmo fario*) ([8]) sections of the future mesoderm from the embryonic ring grafted in this way may give rise to any kind of tissue of the mesoderm and nervous system. In other species, as *Fundulus heteroclitus*, they develop under analogous conditions into an atypical epithelial tissue only. In *Epiplatys fasciolatus* again connexion should be maintained between the anterior sections of the mesoderm of an embryo already formed in order that differentiation of their posterior parts may take place as early as in the stage of the closure of the blastopore ([9]). It may also be assumed that in bony fish the indifferent cells of the embryonic ring, when entering into connexion with the sections of the mesoderm lying in front of them, receive not only mesodermic, but neurogenic factors as well, in so far as it is the mesoderm which gives rise to the latter in the case of induction. If too early isolated from the influence of the mesoderm, the neurogenic agents may overbalance, and the graft will then develop into a neural anlage. In amphibians the ectoderm of the gastrula, which when isolated develops only into an atypical epidermis, transforms into mesoderm if grafted into the region of the future mesoderm. But before doing so it will pass through a stage when there are almost equal tendencies for its developing into neural or mesodermal tissue ([12]). A similar stage may possibly exist in the normal development of certain fish species.

3) Data on the development of the embryonic shield ([8], [9]) of the mesoderm and that of the brain reveal a common feature of these anlages in teleosts: the processes of formation of various types of cells are of less differentiated character than in the rest of vertebrates investigated under this angle.

Institute of Cytology, Histology and Embryology.
Academy of Sciences of the USSR.

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